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A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

INCLUDED IN THE

EXAMINATION QUESTIONS

This volume contains the Keys to the various Examination Questions. These Keys have been so arranged as to be similar in all respects to the Examination Questions to which they refer. The last seven Keys bear the same section numbers as the Examination Questions in the second volume.

CONTENTS.

ARITHMETIC.

(QUESTIONS 1-76.)

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.

605 = Six hundred five.

28,284 = Twenty-eight thousand, two hundred eighty-four.

9,006,042 = Nine millions, six thousand and forty-two.

850,317,002 = Eight hundred fifty millions, three hundred seventeen thousand and two.

700,004 = Seven hundred thousand and four.

(6) Seven thousand six hundred = 7,600.

Eighty-one thousand, four hundred two = 81,402.

Five millions, four thousand and seven = 5,004,007.

One hundred and eight millions, ten thousand and one = 108,010,001.

Eighteen millions and six = 18,000,006.

Thirty thousand and ten = 30,010.

(7) See Art. **71.**

(8) See Art. **76.**

(9) See Art. **73.**

(10) See Art. **73.**

(11) See Art. **74.**

(12) $\frac{13}{8}$ is an improper fraction, since its numerator 13 is greater than its denominator 8.

(13) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(14) To reduce a fraction to its lowest terms means to change its form without changing its value. In order to do this, we must divide both numerator and denominator by the same number until we can no longer find any number (except 1) which will divide both of these terms without a remainder.

To reduce the fraction $\frac{4}{8}$ to its lowest terms, we divide both numerator and denominator by 4, and obtain as a result the fraction $\frac{1}{2}$. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly, $\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$; $\frac{8 \div 4}{32 \div 4} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$; $\frac{32 \div 8}{64 \div 8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$.

(15) When the denominator of any number is not expressed, it is understood to be 1, so that $\frac{6}{1}$ is the same as $6 \div 1$, or 6. To reduce $\frac{6}{1}$ to an improper fraction whose denominator is 4, we must multiply both numerator and denominator by some number which will make the denominator of 6 equal to 4. Since this denominator is 1, by multiplying both terms of $\frac{6}{1}$ by 4, we will have $\frac{6 \times 4}{1 \times 4} = \frac{24}{4}$, which has the *same value* as 6, but has a *different form.*

(16) In order to reduce a mixed number to an improper fraction, we must *multiply the whole number by the denominator of the fraction*, and *add the numerator of the fraction to that product*. This *result* is the *numerator of the improper fraction*, of which the *denominator is the denominator of the fractional part of the mixed number.*

$7\frac{7}{8}$ means the same as $7 + \frac{7}{8}$. In 1 there are $\frac{8}{8}$; hence, in

7 there are $7 \times \frac{8}{8} = \frac{56}{8}$. Add the $\frac{7}{8}$ of the mixed number and we obtain $\frac{56}{8} + \frac{7}{8} = \frac{63}{8}$, which is the required improper fraction.

$$13\frac{5}{16} = \frac{(13 \times 16) + 5}{16} = \frac{213}{16};\ 10\frac{3}{4} = \frac{(10 \times 4) + 3}{4} = \frac{43}{4}.$$

(17) The value of a fraction is obtained by dividing the numerator by the denominator.

To obtain the value of the fraction $\frac{13}{2}$ we divide the numerator 13 by the denominator 2. 2 is contained in 13 six times, with 1 remaining. This 1 remaining is written over the denominator 2, thereby making the fraction $\frac{1}{2}$, which is annexed to the whole number 6, and we obtain $6\frac{1}{2}$ as the mixed number. The reason for performing this operation is the following: In 1 there are $\frac{2}{2}$ (two halves), and in $\frac{13}{2}$ (thirteen halves) there are as many ones (1) as 2 is contained times in 13, which is 6, and $\frac{1}{2}$ (one-half) remaining. Hence, $\frac{13}{2} = 6 + \frac{1}{2} = 6\frac{1}{2}$, the required mixed number.

$$\frac{17}{4} = 4\frac{1}{4};\ \frac{69}{16} = 4\frac{5}{16};\ \frac{16}{8} = 2;\ \frac{67}{64} = 1\frac{3}{64}.$$

(18) A fraction is one or more of the equal parts of a unit, and is expressed by a numerator and a denominator, while a decimal fraction is a number of *tenths*, *hundredths*, *thousandths*, etc., of a unit, and is expressed by placing a period (.) called a decimal point, to the left of the figures of the number, and omitting the denominator.

(19) See Arts. **158** and **159.**

(20) To reduce the fraction $\frac{1}{2}$ to a decimal, we annex one cipher to the numerator, which makes it 1.0. Dividing 1.0, the numerator, by 2, the denominator, gives a quotient of .5, the decimal point being placed before the *one* figure of

the quotient, or .5, since only *one* cipher was annexed to the numerator.

```
 7              5
 -             --
 8)7.000       32)5.00000(.15625
   -----          32
    .875          ---
                  180
                  160        125
                  ---       ----
                   200      1000)125.000(.125
65                 192           1000
---                ---           ----
100)65.00(.65       80            2500
    600             64            2000
    ---             ---           ----
     500            160            5000
     500            160            5000
     ---            ---            ----
```

tenths. hundredths.

(21) . 0 8 = *Eight* **hundredths.**

tenths. hundredths. thousandths.

. 1 3 1 = *One hundred thirty-one* **thousandths.**

tenths. hundredths. thousandths. ten-thousandths.

. 0 0 0 1 = *One* **ten-thousandth.**

tenths. hundredths. thousandths. ten-thousandths. hundred-thousandths. millionths.

0 0 0 0 2 7 = *Twenty-seven* **millionths.**

	tenths.	hundredths.	thousandths.	ten-thousandths.	
.	0	1	0	8	= *One hundred eight* **ten-thousandths.**

	tenths.	hundredths.	thousandths.	ten-thousandths.	
93.	0	1	0	1	=Ninety-three, and *one hundred one* **ten-thousandths.**

In reading decimals, read the number just as you would if there were no ciphers before it. Then count from the decimal point towards the right, beginning with tenths, to as many places as there are figures, and the *name* of the last figure must be annexed to the previous reading of the figures to give the decimal reading. Thus, in the first example given, the simple reading of the figure is *eight*, and the name of its position in the decimal scale is **hundredths**, so that the decimal reading is *eight* **hundredths.** Similarly, the figures in the fourth example are ordinarily read *twenty-seven;* the name of the position of the figure 7 in the decimal scale is **millionths**, giving, therefore, the decimal reading as *twenty-seven* **millionths.**

If there should be a whole number before the decimal point, read it as you would read any whole number, and read the decimal as you would if the whole number were not there; or, read the whole number and then say "and" so many hundredths, thousandths, or whatever it may be, as "ninety-three, and *one* hundred one ten-thousandths."

(22) See Arts. **136** and **141.**

(23) See Art. **148.**

(24) See Art. **155.**

(25) In adding whole numbers, place the numbers to be added directly under each other, so that the extreme right-hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place 9 units under the units column and reserve 1 ten for the column of tens, $8+7+9+1=25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column, and reserve 2 hundreds for the hundreds column. $4+5+2+2=13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column, and reserve the 1 thousand for the thousands column. $2+5+3+1=11$ thousands, or 1 ten-thousand and 1 thousand. Place the 1 thousand in the column of thousands, and reserve the 1 ten-thousand for the column of ten-thousands. $6+1=7$ ten-thousands. Place this seven ten-thousands in the ten-thousands column. There is but one figure, 8, in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

$$\begin{array}{r} 3290 \\ 504 \\ 865403 \\ 2074 \\ 81 \\ 7 \\ \hline 871359 \end{array} \quad \text{Ans.}$$

A simpler (though less scientific) explanation of the same problem is the following: $7+1+4+3+4+0=19$; write the nine and reserve the 1. $8+7+0+0+9+1$ reserved $=25$; write the 5 and reserve the 2. $0+4+5+2+2$ reserved $=13$; write the 3 and reserve the 1. $2+5+3+1$ reserved $=11$; write the 1 and reserve 1. $6+1$ reserved $=7$; write the 7. Bring down the 8 to its place in the sum.

(26)

$$\begin{array}{r} 709 \\ 8304725 \\ 391 \\ 100302 \\ 300 \\ 909 \\ \hline 8{,}407{,}336 \end{array} \quad \text{Ans.}$$

(27) $\frac{1}{8}+\frac{2}{8}+\frac{5}{8}=\frac{1+2+5}{8}=\frac{8}{8}=1.$ Ans.

When the *denominators* of the fractions to be added *are alike*, we know that the units are divided into the *same number of parts* (in this case *eighths*); we, therefore, *add the numerators* of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$, or 1, as the sum.

(28) When the *denominators* are *not alike* we know that the units are divided into *unequal parts*, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators, and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all of these fractions to 16ths and then add their numerators.

$\frac{1}{4}+\frac{3}{8}+\frac{5}{16}=?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms of the fraction by some number which will make the denominator 16. This number evidently is 4, hence, $\frac{1\times 4}{4\times 4}=\frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3\times 2}{8\times 2}=\frac{6}{16}$. The fractions now have a common denominator 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16}+\frac{6}{16}+\frac{5}{16}=\frac{4+6+5}{16}=\frac{15}{16}$. Ans.

(29) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers

separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5 \times 2}{8 \times 2} = \frac{10}{16}$. Adding the two fractional parts of the mixed numbers, we have $\frac{10}{16} + \frac{7}{16} = \frac{10 + 7}{16} = \frac{17}{16} = 1\frac{1}{16}$.

The problem now becomes $42 + 31 + 9 + 1\frac{1}{16} = ?$

Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.

$$\begin{array}{r}
42 \\
31 \\
9 \\
1\frac{1}{16} \\
\hline
83\frac{1}{16}
\end{array} \text{ Ans.}$$

(30) $29\frac{3}{4} + 50\frac{5}{8} + 41 + 69\frac{3}{16} = ?$ $\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}$; $\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$. $\frac{12}{16} + \frac{10}{16} + \frac{3}{16} = \frac{12 + 10 + 3}{16} = \frac{25}{16} = 1\frac{9}{16}$.

The problem now becomes $29 + 50 + 41 + 69 + 1\frac{9}{16} = ?$

$$\begin{array}{rl}
29 & \text{square inches.} \\
50 & \text{square inches.} \\
41 & \text{square inches.} \\
69 & \text{square inches.} \\
1\frac{9}{16} & \text{square inches.} \\
\hline
190\frac{9}{16} & \text{square inches.} \quad \text{Ans.}
\end{array}$$

(31) In addition of decimals, the *decimal points must be placed directly under each other*, so that *tenths* will come *under tenths, hundredths under hundredths, thousandths under thousandths*, etc. The addition is then performed as in whole numbers, *the decimal point of the sum being placed directly under the decimal points above.*

$$\begin{array}{l} .125 \\ .7 \\ .089 \\ .4005 \\ .9 \\ .000027 \\ \hline 2.214527 \quad \text{Ans.} \end{array}$$

(32)

$$\begin{array}{r} 927.416 \\ 8.274 \\ 372.6 \\ 62.07938 \\ \hline 1370.36938 \end{array} \text{ Ans.}$$

(33)

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.

$$\begin{array}{l} .017 \\ .2 \\ .000047 \\ \hline .217047 \end{array}$$

.217047 = *Two hundred and seventeen thousand and forty-seven* **millionths.** Ans.

(34) It will be more convenient to add these numbers together if we express $348\frac{3}{4}$ decimally, since the remaining numbers are decimals. $348\frac{3}{4}$ is equal to 348.75, since $\frac{3}{4}=.75$.

$$\begin{array}{l} 4)3.00(.75 \\ 2\,8 \\ \hline \,20 \\ \,20 \\ \hline \end{array}$$

Since we now have the different weights expressed decimally, we find the total weight of the four lengths of shafting to be 316.5 lb. + 402.3 lb. + 348.75 lb. + 309.4 lb., or 1,376.95 lb. Ans.

316.5	lb.
402.3	lb.
348.75	lb.
309.4	lb.
1376.95	lb. Ans.

(35) If the cost of the coal consumed by a nest of steam boilers amounts to $15.83 on Monday, to $14.70 on Tuesday, to $14.28 on Wednesday, to $13.87 on Thursday, to $14.98 on Friday, and to $12.65 on Saturday, then, we find the total cost of the week's supply by adding the different amounts together; hence, $15.83 + $14.70 + $14.28 + $13.87 + $14.98 + $12.65 = $86.31.

$15.83
14.70
14.28
13.87
14.98
12.65
$86.31 Ans.

(36) The steam engine, during the 12-hour test, showed that the number of revolutions made were 150,508, since 12,600 + 12,444 + 12,467 + 12,528 + 12,468 + 12,590 + 12,610 + 12,589 + 12,576 + 12,558 + 12,546 + 12,532 = 150,508 rev. Ans.

12600	revolutions.
12444	revolutions.
12467	revolutions.
12528	revolutions.
12468	revolutions.
12590	revolutions.
12610	revolutions.
12589	revolutions.
12576	revolutions.
12558	revolutions.
12546	revolutions.
12532	revolutions.
150508	revolutions. Ans.

(37) In subtracting whole numbers, place the subtrahend, or smaller number, under the minuend, or larger number, so that the right-hand figures stand directly under each other. Begin *at the right* to subtract.

$$\begin{array}{r} (a)\quad 50962 \\ 3338 \\ \hline 47624 \end{array} \text{ Ans.}$$

We can not subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. 1 *ten* = 10 *units*, so we have 10 units + 2 units = 12 units. Then, 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so only 5 tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We can not subtract 3 thousands from 0 thousands, so we take 1 ten-thousand from 5 ten-thousands and add it to the 0 thousands. 1 *ten-thousand* = 10 *thousands*, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten-thousand from 5 ten-thousands and have 4 ten-thousands remaining. Since there are no ten-thousands in the subtrahend, the 4 in the ten-thousands column in the minuend is brought down into the same column in the remainder, because 0 from 4 leaves 4.

$$\begin{array}{r} (b)\quad 15339 \\ 10001 \\ \hline 5338 \end{array} \text{ Ans.}$$

(38)

$$\begin{array}{r} (a)\quad 70968 \\ 32975 \\ \hline 37993 \end{array} \text{ Ans.} \qquad \begin{array}{r} (b)\quad 100000 \\ 98735 \\ \hline 1265 \end{array} \text{ Ans.}$$

(39) $\frac{7}{8} - \frac{7}{16} = ?$ When the *denominators* of fractions are *not alike*, it is evident that the units are divided into *unequal parts*, therefore, before subtracting, *reduce the fractions to fractions having a common denominator*. Then, *subtract the numerators, and place the remainder over the common denominator.*

$$\frac{7 \times 2}{8 \times 2} = \frac{14}{16}. \quad \frac{14}{16} - \frac{7}{16} = \frac{14 - 7}{16} = \frac{7}{16}. \quad \text{Ans.}$$

$13 - 7\frac{7}{16} = ?$ This problem may be solved in two ways:

First: $13 = 12\frac{16}{16}$, since $\frac{16}{16} = 1$, and $12\frac{16}{16} = 12 + \frac{16}{16} = 12 + 1 = 13$.

$$\begin{array}{r} 12\frac{16}{16} \\ 7\frac{7}{16} \\ \hline 5\frac{9}{16} \end{array}$$

We can now subtract the whole numbers separately, and the fractions separately, and obtain $12 - 7 = 5$ and $\frac{16}{16} - \frac{7}{16} = \frac{16 - 7}{16} = \frac{9}{16}$. $5 + \frac{9}{16} = 5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$$13 = \frac{13}{1} = \frac{13 \times 16}{1 \times 16} = \frac{208}{16}. \quad 7\frac{7}{16} = \frac{(7 \times 16) + 7}{16} = \frac{112 + 7}{16} = \frac{119}{16}.$$

Subtracting, we have $\frac{208}{16} - \frac{119}{16} = \frac{208 - 119}{16} = \frac{89}{16}$ and $\frac{89}{16} =$

$$\begin{array}{r} 16)89(5\frac{9}{16} \\ 80 \\ \hline 9 \\ \hline 16 \end{array}$$

the same result that was obtained by the first method.

$312\frac{9}{16} - 229\frac{5}{32} = ?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this, we have $\frac{9}{16} = \frac{9 \times 2}{16 \times 2} = \frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312 - 229 = 83$ and $\frac{18}{32} - \frac{5}{32} = \frac{18 - 5}{32} = \frac{13}{32}$.

$$\begin{array}{r} 312\frac{18}{32} \\ 229\frac{5}{32} \\ \hline 83\frac{13}{32} \end{array} \quad 83 + \frac{13}{32} = 83\frac{13}{32}. \quad \text{Ans.}$$

(40) (*a*) In subtraction of decimals, *place the decimal points directly under each other*, and proceed as in the subtraction of whole numbers, placing the *decimal point in the remainder directly under the decimal points above.*

$$\begin{array}{r} 709.6300 \\ .8514 \\ \hline 708.7786 \end{array} \quad \text{Ans.}$$

In the above example, we proceed as follows: We can not

subtract 4 ten-thousandths from 0 ten-thousandths, and as there are no thousandths, we take 1 hundredth from the three hundredths. 1 *hundredth* = 10 *thousandths* = 100 *ten-thousandths*. 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 *thousandths* + 6 *ten-thousandths*. Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths leaves but 2 hundredths now.) Since we can not do this, we take 1 tenth from 6 tenths. 1 tenth = 10 hundredths, and 10 hundredths + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this can not be done, we take 1 unit from the 9 units. 1 *unit* = 10 *tenths*. 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the tenths place in the remainder. In the minuend we now have 708 units (one unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units, hence, we write 708 in the remainder.

(*b*)		(*c*)		(*d*)	
81.963		18.00		1.000	
1.700		.18		.001	
80.263	Ans.	17.82	Ans.	.999	Ans.

(*e*) 872.1 − (.8721 + .008) = ? In this problem we are to subtract (.8721 + .008) from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

$$\begin{array}{r} .8721 \\ .0080 \\ \hline .8801 \end{array} \textit{ sum}$$

Subtracting the sum (obtained by adding the decimals enclosed within the parenthesis) from the number 872.1 (as required by the minus sign before the parenthesis), we obtain the required remainder.

$$\begin{array}{r} 872.1000 \\ .8801 \\ \hline 871.2199 \end{array} \text{ Ans.}$$

(*f*) $(5.028 + .0073) - (6.704 - 2.38) = ?$ First perform the operations as indicated by the signs between the numbers enclosed by the parentheses. The first parenthesis shows that 5.028 and .0073 are to be added. This gives 5.0353 as their sum.

$$\begin{array}{r} 5.0280 \\ .0073 \\ \hline 5.0353 \end{array} \textit{ sum.}$$

$$\begin{array}{r} 6.704 \\ 2.380 \\ \hline 4.324 \end{array} \textit{ diff.}$$

The second parenthesis shows that 2.38 is to be subtracted from 6.704. The difference is found to be 4.324.

The sign between the parentheses indicates that the quantities obtained by performing the above operations are to be subtracted, namely, that 4.324 is to be subtracted from 5.0353. Performing this operation, we obtain .7113 as the final result.

$$\begin{array}{r} 5.0353 \\ 4.3240 \\ \hline .7113 \end{array} \text{ Ans.}$$

(41) In subtracting a decimal from a fraction, or subtracting a fraction from a decimal, either reduce the fraction to a decimal before subtracting, or reduce the decimal to a fraction and then subtract.

(*a*) $\frac{7}{8} - .807 = ?$ $\frac{7}{8}$ reduced to a decimal becomes

$$\begin{array}{r} 7 \\ 8\overline{)7.000} \\ .875 \end{array}$$

$$\begin{array}{r} .875 \\ .807 \\ \hline .068 \end{array} \text{ Ans.}$$

Subtracting .807 from .875 the remainder is .068, as shown.

(*b*) $.875 - \frac{3}{8} = ?$ Reducing .875 to a fraction we have $.875 = \frac{875}{1{,}000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; hence, $\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{1}{2}$ or .5. Ans.

Or, by reducing $\frac{3}{8}$ to a decimal, $\frac{3}{8})\underline{3.000}$ and then sub-
$.375$

tracting, we obtain $.875 - .375 = .5 = \frac{5}{10} = \frac{1}{2}$, the same answer as above.

$$\begin{array}{r} .875 \\ \underline{.375} \\ .500 \end{array} \quad \text{Ans.}$$

(*c*) $\left(\frac{5}{32} + .435\right) - \left(\frac{21}{100} - .07\right) = ?$ We first perform the operations as indicated by the signs between the numbers enclosed by the parentheses. Reducing $\frac{5}{32}$ to a decimal, we obtain $\frac{5}{32} = .15625$ (see example 20).

Adding .15625 and .435, $\qquad \frac{21}{100} = .21$; subtracting,

$$\begin{array}{l} .15625 \\ \underline{.435} \\ .59125 \end{array} \qquad\qquad \begin{array}{l} .21 \\ \underline{.07} \\ .14 \textit{ diff.} \end{array}$$

We are now prepared to perform the operation indicated by the minus sign between the parentheses, which is,

$$\begin{array}{l} .59125 \\ \underline{.14} \\ .45125 \textit{ diff.} \end{array}$$

Ans.

(*d*) This problem means that 33 millionths and 17 thousandths are to be added. Also, that 53 hundredths and 274 thousandths are to be added, and the smaller of these sums is to be subtracted from the larger sum. Thus, $(.53 + .274) - (.000033 + .017) = ?$

tenths.	hundredths.	thousandths.	
.5	3		
.2	7	4	
.8	0	4	*sum.*

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	
.0	0	0	0	3	3	
.0	1	7				
.0	1	7	0	3	3	*sum.*

$$\begin{array}{ll} .804 & \textit{larger sum.} \\ \underline{.017033} & \textit{smaller sum.} \\ .786967 & \textit{diff.} \quad \text{Ans.} \end{array}$$

(42) $482\frac{4}{5} + 316\frac{1}{3} + 390\frac{3}{4} =$ what?

When mixed numbers are to be added, add the fractional parts of the mixed numbers separately, and, if the resulting

fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction. First, we will reduce the fractional parts $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$ to equivalent fractions having the least common denominator. In this case the least common denominator equals the product of the denominators 5, 3, and 4, since we can not divide any two of them by any number (except 1) without having a remainder, as can be done in the examples in Art. **96.** Hence, the least common denominator $= 5 \times 3 \times 4 = 60$. Reducing $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$ to fractions having this least common denominator, we have 60 divided by the first denominator 5, equals 12. Then, $\frac{4}{5} \times \frac{12}{12} = \frac{48}{60}$. 60 divided by the second denominator 3, equals 20. Then, $\frac{1}{3} \times \frac{20}{20} = \frac{20}{60}$. 60 divided by the third denominator 4, equals 15. Then, $\frac{3}{4} \times \frac{15}{15} = \frac{45}{60}$. The sum of these fractions equals

$$\frac{48}{60} + \frac{20}{60} + \frac{45}{60} = \frac{48 + 20 + 45}{60} = \frac{113}{60}, \text{ or } 1\frac{53}{60}.$$

The problem now becomes $482 + 316 + 390 + 1\frac{53}{60}$, the sum of which equals $1{,}189\frac{53}{60}$.

$$\begin{array}{r} 482 \\ 316 \\ 390 \\ 1\frac{53}{60} \\ \hline 1189\frac{53}{60} \end{array}$$

$$\begin{array}{ll} 1500 & \text{represents the actual horsepower required.} \\ 1189\frac{53}{60} & \text{represents the indicated horsepower of the engines in use.} \\ \hline 310\frac{7}{60} \text{ or } 310.11\frac{2}{3} & = \text{the H. P. to be developed by the new engine. Ans.} \end{array}$$

$\frac{7}{60}$ reduced to its equivalent decimal $= \frac{7}{60}$

```
60) 7.00 (.11⅔
    60
    100
     60
     40/60 = 2/3.
```

(43)

3040 = No. of gallons in the tank at the beginning of the day.
4780 = No. of gallons pumped in during the morning.
7820 = No. of gallons in the tank after 4,780 gallons were added.
7240 = No. of gallons drawn out during the morning.
580 = No. of gallons in the tank at the beginning of the afternoon.
8675 = No. of gallons pumped in during the afternoon.
9255 = No. of gallons in the tank after 8,675 gallons were added.
7633 = No. of gallons drawn out during the afternoon.
1622 = No. of gallons remaining in the tank at night. Ans.

(44) The height of the stack equals 45 feet. Since six of the plates which make this stack are 7 feet long, the length of the six plates equals 6×7 or 42 feet. However, there is an allowance of 15 inches, or 1 foot 3 inches, for lapping, which, deducted from 42 feet, makes a total length of 40 feet 9 inches. Since the total height of the stack is 45 feet, the length of the seventh plate must equal 45 feet—40 feet 9 inches, or 4 ft. 3 in. Ans.

45 feet 0 inches
40 feet 9 inches
4 feet 3 inches

(45) Since the inside diameter of the steam pipe is 6.06 inches, and the outside diameter is 6.62 inches, there is a difference of 6.62 — 6.06, or .56 of an inch, in both diameters.

But .56 of an inch is just twice the thickness of the pipe; hence, the pipe is $\frac{1}{2}$ of .56, or .28 of an inch thick.

(46) In the multiplication of whole numbers, place the multiplier under the multiplicand, and multiply each term of the multiplicand by each term of the multiplier, writing the right-hand figure of each product obtained under the term of the multiplier which produces it.

(*a*)
$$\begin{array}{r} 526,387 \\ 7 \\ \hline 3,684,709 \end{array} \text{ Ans.}$$

7 times 7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 + 4 tens reserved = 60 tens, or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7×3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve the 2 thousands. 7×6 thousands = 42 thousands; 42 + 2 thousands reserved = 44 thousands, or 4 ten-thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten thousands. 7×2 ten-thousands = 14 ten-thousands; 14 + 4 ten-thousands reserved = 18 ten-thousands, or 1 hundred-thousand and 8 ten-thousands. Write the 8 ten-thousands and reserve the 1 hundred-thousand. 7×5 hundred-thousands = 35 hundred-thousands; 35 + 1 hundred-thousand reserved = 36 hundred-thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred-thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times 7 = 49; write the 9 and reserve the 4. 7 times 8 = 56; 56 + 4 reserved = 60; write the 0 and reserve the 6. 7 times 3 = 21; 21 + 6 reserved = 27; write the 7 and reserve the 2. $7 \times 6 = 42$; 42 + 2 reserved = 44; write the 4 and reserve 4. $7 \times 2 = 14$; 14 + 4 reserved = 18; write the 8 and reserve 1. $7 \times 5 = 35$; **35 + 1 reserved = 36; write the 36.**

(*b*)

```
  700,298
       17
 --------
  4902086
 700298
---------
11,905,066 Ans.
```

In this case the multiplier is 17 *units*, or 1 *ten* and 7 *units*, so that the product is obtained by adding two partial products, namely, $7 \times 700,298$ and $10 \times 700,298$. The actual operation is performed as follows:

7 times $8 = 56$; write the 6 and reserve the 5. 7 times $9 = 63$; $63 + 5$ reserved $= 68$; write the 8 and reserve the 6. 7 times $2 = 14$; $14 + 6$ reserved $= 20$; write the 0 and reserve the 2. 7 times $0 = 0$; $0 + 2$ reserved $= 2$; write the 2. 7 times $0 = 0$; $0 + 0$ reserved $= 0$; write the 0. 7 times $7 = 49$; $49 + 0$ reserved $= 49$; write the 49.

To multiply by the 1 ten we have 1 ten times 8 units $=$ 8 tens and 0 units. We do not write the 0 units, but write the 8 tens under the 8 tens in the first partial product above. We next multiply 1 ten, and 9 tens $= 90$ tens $= 9$ hundreds $+ 0$ tens; write the 9 hundreds under the 0 hundreds of the first partial product above. Again, 1 ten times 2 hundreds $= 2$ thousands; write the 2 thousands under the 2 thousands above. 1 ten times 0 thousands $= 0$ ten-thousands; write the 0 in the ten-thousands place. 1 ten times 0 ten-thousands $= 0$ hundred-thousands; write the 0 in the hundred-thousands place. 1 ten times 7 hundred-thousands $= 7$ millions; write the 7 in the millions place. This completes the second partial product. Add the two partial products; their sum equals the entire product.

(*c*)

```
      217
      103
     ----
      651
     000
    217
   ------
    22351
       67
  -------
   156457
  134106
---------
1,497,517 Ans.
```

Multiply any two of the numbers together, and multiply their product by the third number.

(47) If your watch ticks every second, to find how many times it ticks in one week it is necessary to find the number of seconds in 1 week.

```
     60 seconds = 1 minute.
     60 minutes = 1 hour.
  -------
  3,600 seconds = 1 hour.
     24 hours = 1 day.
  -------
  14400
  7200
  -------
 86,400 seconds = 1 day.
      7 days = 1 week.
  -------
604,800 seconds in one week, or the number of times that
  Ans.  your watch ticks in a week.
```

(48) (*a*) There are 3 decimal places in the multiplicand and 3 in the multiplier; hence, there are 3 + 3 or 6 decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary six decimal places.

```
   .107
   .013
   ----
    321
   107
 -------
.001391  Ans.
```

(*b*) There are 2 decimal places in the multiplier and none in the multiplicand; hence, there are 2 + 0 or 2 decimal places in the first product.

Since in the second multiplication there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are 3 + 2 or 5 decimal places in the second product.

When there are one or more ciphers in the multiplier, multiply just the same as with the other figures.

```
(b)    203
      2.03
     -----
       609
      000
     406
    ------
    412.09
      .203
  --------
    123627
   00000
  82418
  --------
  83.65427  Ans.
```

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parentheses, and then whatever may be required by the sign between the parentheses.

```
 31.85
   2.7
 ------
 22295
 6370
 ------
 85.995
```

The first parenthesis shows that the numbers 2.7 and 31.85 are to be multiplied together.

The second parenthesis shows that 316 is to be taken from 3.16.

```
3.160
 .316
-----
2.844
```

The product obtained by performing the operation indicated by the signs within the first parenthesis is now multiplied by the remainder obtained by performing the operation indicated by the signs within the second parenthesis.

```
      85.995
       2.844
    --------
     343980
    343980
   687960
  171990
  -----------
  244.569780  Ans.
```

(*d*) $(107.8 + 6.541 - 31.96) \times 1.742 = ?$

```
   107.8               82.381
 +   6.541           ×  1.742
 -------            ---------
   114.341            164762
 -  31.96            329524
 -------            576667
    82.381          82381
                    ----------
                    143.507702   Ans.
```

(49) If an engine and boiler are worth \$3,246, and the building is worth 3 times as much plus \$1,200, then the building is worth

```
        $3246
            3 times
        -----
         9738
   plus  1200
        ------
       $10938 = value of building.
```

If the tools are worth twice as much as the building, plus \$1,875, then the tools are worth

$$
\begin{array}{rr}
 & \$10938 \\
 & 2 \\
\hline
 & 21876 \\
\text{plus} & 1875 \\
\hline
 & \$23751 = \text{value of tools.}
\end{array}
$$

$$
\begin{array}{rr}
\text{Value of building} = & \$10938 \\
\text{Value of tools} \quad = & 23751 \\
\hline
 & \$34689 = \text{value of the building and tools. } (a) \text{ Ans.}
\end{array}
$$

$$
\begin{array}{rr}
\text{Value of engine and boiler} = & \$3246 \\
\text{Value of building and tools} = & 34689 \\
\hline
 & \$37935 = \text{value of the whole plant. } (b) \text{ Ans.}
\end{array}
$$

(50) If one 3-inch tube measures $15\frac{1}{2}$ ft. in length, 60 of these tubes would measure $60 \times 15\frac{1}{2}$ ft., or 930 ft. in length. If 1 foot of tubing has a heating surface of .728 sq. ft., then it is evident that 930 ft. of tubing would have a heating surface of $930 \times .728$ ft., or 677.04 sq. ft.

$$
\begin{array}{r}
15\frac{1}{2} \\
60 \\
\hline
900 \\
30 \\
\hline
930 \text{ ft.}
\end{array}
$$

$$
\begin{array}{r}
.728 \\
930 \\
\hline
21840 \\
6552 \\
\hline
677.040 \text{ sq. ft.}
\end{array}
$$

(51) If in 1 hour 10 pounds of coal are burned for every square foot of grate area, and 9 pounds of water are evaporated for every pound of coal burned, then in 1 hour there would be 9×10 or 90 pounds of water evaporated for 1 sq. ft. of grate area; and since the grate area is 30 sq. ft., the amount of water evaporated would be $30 \times 90 = 2,700$ lb. Since 2,700 lb. of water are evaporated in 1

hour, in a day of 10 hours there would be 10 × 2,700 lb., or 27,000 lb. of water evaporated.

(52) Each stroke of the engine is 18 inches in length. Since the piston makes 2 strokes for each revolution, it would pass over a distance of 2 × 18 inches = 36 inches, or 3 feet in 1 revolution, and in making 160 revolutions it would pass over 160 × 3, or 480 ft. Since 480 ft. are passed over in 1 minute, in 1 hour, or 60 minutes, the distance passed over equals 60 × 480 = 28,800 ft. Since the steam engine runs 6 days a week and $8\frac{1}{2}$ hours per day, the total number of hours it runs per week $= 6 \times 8\frac{1}{2}$, or 51 hours. If the piston passes over a distance of 28,800 ft. in 1 hour, in 51 hours it would pass over 51 × 28,800 ft., or 1,468,800 ft. Ans.

$$\begin{array}{r} 480\text{ ft.} \\ \times\ 60 \\ \hline 28800\text{ ft.} \end{array}$$

$$\begin{array}{r} 28800 \\ \times\ \ \ 51 \\ \hline 28800 \\ 144000\ \ \\ \hline 1468800 \end{array}$$

(53) Since the first pump delivers $1\frac{1}{4}$ gallons for each stroke, and runs at the rate of 75 strokes per minute, then the number of gallons delivered by the first pump equals $75 \times 1\frac{1}{4} = 75 \times 1.25 = 93.75$ gallons.

$$\begin{array}{r} 1.25 \\ \times\ \ 75 \\ \hline 625 \\ 875\ \ \\ \hline 93.75 \end{array}$$

$\frac{7}{8} = .875$, since

$$\begin{array}{r} 8\overline{)7.000} \\ .875 \end{array}$$

The second pump delivers $\frac{7}{8}$ or .875 of a gallon for each stroke, and runs at the rate of 115 strokes per minute; then the total number of gallons delivered by the second pump equals 115 × .875 = 100.625 gallons.

$$\begin{array}{r} .875 \\ \times\ 115 \\ \hline 4375 \\ 875\ \ \\ 875\ \ \ \ \\ \hline 100.625 \end{array}$$

```
8)5.000
   .625

  1.625
×    96
   9750
 14625
156.000

 350.375
      60
21022.500
```

The third pump delivers $1\frac{5}{8}$, or 1.625 (since $\frac{5}{8} = .625$) gallons, for each stroke, and runs at the rate of 96 strokes per minute; then, the total number of gallons delivered by the third pump equals $96 \times 1.625 = 156$ gallons.

The total number of gallons delivered by the three pumps in one minute equals $93.75 + 100.625 + 156$ gallons, or 350.375 gallons. Since there are 60 minutes in 1 hour, we find that the total number of gallons of water fed to the boilers in 1 hour equals $60 \times 350.375 = 21,022.5$. Ans.

(54) (*a*) 84)962842.0000(11462.4047 + Ans.

```
84)962842.0000(11462.4047+
   84
   122
    84
    388
    336
     524
     504
      202
      168
       340
       336
        400
        336
         640
         588
          52
```

84 is contained in 96, once. Place 1 as the first figure in the quotient and multiply the divisor 84 by it. Subtract

the product, which is 84, from 96, leaving a remainder of 12. Bring down the next figure in the dividend, which is 2, annex it to 12, making a new dividend of 122.

84 is contained in 122 once. Place 1 as the second figure in the quotient and multiply the divisor 84 by it. Subtract the product (84) from 122, leaving a remainder of 38. Bring down the next figure in the dividend, which is 8, annex it to 38, making a new dividend of 388.

84 is contained in 388 4 times. Place 4 as the third figure in the quotient and multiply the divisor 84 by it. The product is 336. Subtract the product from 388, leaving a remainder of 52. Bring down the next figure, which is 4, annex it to 52, making a new dividend of 524.

84 is contained in 524 6 times. Place 6 as the fourth figure in the quotient. Multiply the divisor 84 by it and subtract the product (504) from 524, leaving a remainder of 20. Bring down the next figure, which is 2, annex it to 20. making a new dividend of 202.

84 is contained in 202 2 times. Place 2 as the fifth figure in the quotient. Multiply the divisor 84 by it, and subtract the product (168) from 202, leaving a remainder of 34. If it is desired to carry the quotient to 4 decimal places, annex 4 ciphers to the dividend and continue in the same way. In the quotient point off as many decimal places as there are ciphers annexed, or, in this case, four decimal places.

```
(b) 63)39728.000(630.603+ Ans.
       378
       ---
        192
        189
        ----
          380*
          378
          ---
            200
            189
            ---
             11
```

*63 is not contained in 38, so we place a cipher in the quotient and bring down the next figure in the dividend, which is a cipher that has been annexed. making a new dividend of 380.

63 is contained in 380 6 times. $6 \times 63 = 378$. Subtracting

378 from 380 leaves 2. Bringing down the next figure in the dividend we have 20 for a new dividend. 63 is not contained in 20, so we place a cipher in the quotient and bring down the next cipher in the dividend, making a new dividend of 200. 63 is contained 3 times in 200.

```
(c) 108)29714.0000(275.1296  Ans.
        216
        ---
         811   (d) 135)406089.0000(3008.0666
         756           405                 Ans.
         ---           ---
          554          *1089
          540           1080
          ---           ----
           140             900
           108             810
           ---             ---
            320             900
            216             810
            ---             ---
            1040             900
             972             810
             ---             ---
              680             90
              648
              ---
               32
```

*135 is not contained in 10, so we place a cipher as the second figure in the quotient. Bringing down the next figure 8 and annexing it to 10, we have a new dividend of 108. 135 is not contained in 108, so we place a cipher as the third figure in the quotient, and bring down the next figure in the dividend, or 9, and annex it to 108, making a new dividend of 1,089. 135 is contained in 1,089 8 times. Write 8 as the fourth figure in the quotient. Multiply 135 by 8 and subtract the product (1,080) from 1,089, which leaves a remainder of 9. Bring down the next figure in the dividend, which is a cipher that has been annexed, annex it to the remainder 9, making a new dividend of 90. As 135 is not contained in 90, we place a 0 in the quotient and bring down another cipher from the dividend, making a new dividend of 900. 135 is contained in 900 6 times

Write 6 as the next figure in the quotient, multiply 135 by 6 and subtract the product (810) from 900, which leaves a remainder of 90. Bring down the next figure (0) in the dividend, annex it to the remainder 90, making a new dividend of 900. 135 is contained in 900 six times. Place 6 as the next figure in the quotient and multiply the divisor by it. It is plain that each succeeding figure of the quotient will be 6. Point off *four* decimal places in the quotient, since four ciphers were annexed.

(55) In division of fractions, ***invert the divisor*** (or, in other words, turn it upside down) ***and proceed as in multiplication.***

(*a*) $35 \div \frac{5}{16} = \frac{35}{1} \times \frac{16}{5} = \frac{35 \times 16}{1 \times 5} = \frac{560}{5} = 112.$ Ans.

(*b*) $\frac{9}{16} \div 3 = \frac{9}{16} \div \frac{3}{1} = \frac{9}{16} \times \frac{1}{3} = \frac{9 \times 1}{16 \times 3} = \frac{9}{48} = \frac{3}{16}.$ Ans.

(*c*) $\frac{17}{2} \div 9 = \frac{17}{2} \div \frac{9}{1} = \frac{17}{2} \times \frac{1}{9} = \frac{17 \times 1}{2 \times 9} = \frac{17}{18}.$ Ans.

(*d*) $\frac{113}{64} \div \frac{7}{16} = \frac{113}{64} \times \frac{16}{7} = \frac{113 \times 16}{64 \times 7} = \frac{1{,}808}{448} =$

$$\frac{452}{112} = \frac{113}{28}) \, 113 \, (4\tfrac{1}{28}. \quad \text{Ans.}$$
$$\begin{array}{r} 112 \\ \hline 1 \end{array}$$

(*e*) $15\frac{3}{4} \div 4\frac{3}{8} = ?$ Before proceeding with the division, reduce both of the mixed numbers to improper fractions. Thus, $15\frac{3}{4} = \frac{(15 \times 4) + 3}{4} = \frac{60+3}{4} = \frac{63}{4}$, and $4\frac{3}{8} = \frac{(4 \times 8) + 3}{8} = \frac{32 + 3}{8} = \frac{35}{8}$. The problem is now $\frac{63}{4} \div \frac{35}{8} = ?$ As before, invert the divisor and multiply; $\frac{63}{4} \div \frac{35}{8} = \frac{63}{4} \times \frac{8}{35} = \frac{63 \times 8}{4 \times 35} = \frac{504}{140} = \frac{252}{70} = \frac{126}{35} = \frac{18}{5}$.

$$\frac{18}{5}) \, 18 \, (3\tfrac{3}{5}. \quad \text{Ans.}$$
$$\begin{array}{r} 15 \\ \hline 3 \end{array}$$

(56) (*a*) $.875 \div \frac{1}{2} = .875 \div .5 \left(\text{since } \frac{1}{2} = .5\right) = 1.75$. Ans.
Another way of solving this is to reduce .875 to its equivalent common fraction and then divide.

$.875 = \frac{7}{8}$, since $.875 = \frac{875}{1,000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; then, $\frac{7}{8} \div \frac{1}{2} = \frac{7}{8} \times \frac{2}{1} = \frac{7 \times 2}{8} = \frac{14}{8} = \frac{7}{4} = 1\frac{3}{4}$.

Since $\frac{3}{4} = $

$$\begin{array}{r|l} 4) & 3.00(.75 \\ & \underline{2\,8} \\ & 20 \\ & \underline{20} \end{array}$$

$1\frac{3}{4} = 1.75$, the same answer as above.

(*b*) $\frac{7}{8} \div .5 = \frac{7}{8} \div \frac{1}{2} \left(\text{since } .5 = \frac{1}{2}\right) = \frac{7}{8} \times \frac{2}{1} = \frac{7 \times 2}{8} = \frac{14}{8} = \frac{7}{4} = 1\frac{3}{4}$, or 1.75. Ans.

This can also be solved by reducing $\frac{7}{8}$ to its equivalent decimal and dividing by .5; $\frac{7}{8} = .875$; $.875 \div .5 = 1.75$. Since there are three decimal places in the dividend and one in the divisor, there are $3 - 1$, or 2 decimal places in the quotient.

(*c*) $\frac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125} = ?$ We will solve this problem by first reducing the decimals to their equivalent common fractions.

$.375 = \frac{375}{1,000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$. $\frac{3}{8} \times \frac{1}{4} = \frac{3}{32}$, or the value of the numerator of the fraction.

$.125 = \frac{125}{1,000} = \frac{25}{200} = \frac{1}{8}$. Reducing $\frac{1}{8}$ to 16ths, we have $\frac{1 \times 2}{8 \times 2} = \frac{2}{16}$. Then, $\frac{5}{16} - \frac{2}{16} = \frac{3}{16}$, or the value of the

denominator of the fraction. The problem is now reduced to

$$\frac{\frac{3}{32}}{\frac{3}{16}} = ? \quad \frac{\frac{3}{32}}{\frac{3}{16}} = \frac{3}{32} \div \frac{3}{16} = \frac{3}{32} \times \frac{16}{3} = \frac{48}{96} = \frac{1}{2} \text{ or } .5. \quad \text{Ans.}$$

(57) (*a*) $(72 \times 48 \times 28 \times 5) \div (84 \times 15 \times 7 \times 6)$.

Placing the numerator over the denominator the problem becomes

$$\frac{72 \times 48 \times 28 \times 5}{84 \times 15 \times 7 \times 6} = ?$$

The 5 in the numerator and 15 in the denominator are both *divisible* by 5, since 5 divided by 5 equals 1, and 15 divided by 5 equals 3. *Cross off* the 5 and write the 1 *over* it; also *cross off* the 15 and write the 3 *under* it. Thus,

$$\frac{72 \times 48 \times 28 \times \overset{1}{\cancel{5}}}{84 \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

72 in the numerator and 84 in the denominator are *divisible* by 12, since 72 divided by 12 equals 6, and 84 divided by 12 equals 7. *Cross off* the 72 and write the 6 *over* it; also, *cross off* the 84 and write the 7 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times 28 \times \overset{1}{\cancel{5}}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

Again, 28 in the numerator and 7 in the denominator are *divisible* by 7, since 28 divided by 7 equals 4, and 7 divided by 7 equals 1. *Cross off* the 28 and write the 4 *over* it; also, *cross off* the 7 and write the 1 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times \underset{1}{\cancel{7}} \times 6} =$$

Again, 48 in the numerator and 6 in the denominator are *divisible* by 6, since 48 divided by 6 equals 8, and 6 divided by 6 equals 1. *Cross off* the 48 and write the 8 *over* it; also, *cross off* the 6 and write the 1 *under* it. Thus,

$$\frac{\overset{6}{\cancel{12}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{6}}} =$$

Again, 6 in the numerator and 3 in the denominator are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and write the 2 *over* it; also, *cross off* the 3 and write the 1 *under* it. Thus

$$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{12}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{7}{\cancel{84}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{15}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{6}}}.$$

Since there are *no two remaining numbers* (one in the numerator and one in the denominator) *divisible* by *any number* except 1, without a remainder, it is *impossible* to cancel further.

Multiply all the *uncanceled numbers* in the numerator together, and divide their *product* by the *product* of all the *uncanceled* numbers in the denominator. The *result* will be the *quotient*. The *product* of all the *uncanceled numbers* in the numerator equals $2 \times 8 \times 4 \times 1 = 64$; the product of all the *uncanceled* numbers in the denominator equals $7 \times 1 \times 1 \times 1 = 7$.

Hence, $$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{12}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{7}{\cancel{84}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{15}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{6}}} = \frac{2 \times 8 \times 4 \times 1}{7 \times 1 \times 1 \times 1} = \frac{64}{7} = 9\frac{1}{7}. \text{ Ans.}$$

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Placing the numerator over the denominator, the problem becomes
$$\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$$

The 50 in the numerator and 70 in the denominator are both *divisible* by 10, since 50 divided by 10 equals 5, and 70 divided by 10 equals 7. *Cross off* the 50 and write the 5 *over* it; also, *cross off* the 70 and write the 7 *under* it. Thus,

$$\frac{80 \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times 20} =$$

Also, 80 in the numerator and 20 in the denominator are *divisible* by 20, since 80 divided by 20 equals 4, and 20 divided by 20 equals 1. *Cross off* the 80 and write the 4 *over* it; also, cross off the 20 and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times \underset{1}{\cancel{20}}} =$$

Again, 16 in the numerator and 24 in the denominator are *divisible* by 8, since 16 divided by 8 equals 2, and 24 divided by 8 equals 3. *Cross off* the 16 and write the 2 *over* it; also, cross off the 24 and write the 3 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times 14}{\underset{7}{\cancel{70}} \times 50 \times \underset{3}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

Again, 60 in the numerator and 50 in the denominator are *divisible* by 10, since 60 divided by 10 equals 6, and 50 divided by 10 equals 5. *Cross off* the 60 and write the 6 *over* it; also, cross off the 50 and write the 5 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times 14}{\underset{7}{\cancel{70}} \times \underset{5}{\cancel{50}} \times \underset{3}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

The 14 in the numerator and 7 in the denominator are *divisible* by 7, since 14 divided by 7 equals 2, and 7 divided by 7 equals 1. *Cross off* the 14 and write the 2 *over* it; also, cross off the 7 and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}} \times \underset{5}{\cancel{50}} \times \underset{3}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

The 5 in the numerator and 5 in the denominator are *divisible* by 5, since 5 divided by 5 equals 1. *Cross off* the

5 of the *dividend* and write the 1 *over* it; also, cross off the 5 of the *divisor* and write the 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{\overset{1}{\cancel{5}}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}} \times \underset{\underset{1}{\cancel{5}}}{\cancel{50}} \times \underset{\underset{1}{3}}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

The 6 in the numerator and 3 in the denominator are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and place 2 *over* it; also, cross off the 3 and place 1 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{\overset{2}{\cancel{6}}}{\cancel{60}} \times \overset{\overset{1}{\cancel{5}}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}} \times \underset{\underset{1}{\cancel{5}}}{\cancel{50}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{24}} \times \underset{1}{\cancel{20}}} =$$

Hence, $\frac{\overset{4}{\cancel{80}} \times \overset{\overset{2}{\cancel{6}}}{\cancel{60}} \times \overset{\overset{1}{\cancel{5}}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}} \times \underset{\underset{1}{\cancel{5}}}{\cancel{50}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{24}} \times \underset{1}{\cancel{20}}} = \frac{4 \times 2 \times 1 \times 2 \times 2}{1 \times 1 \times 1 \times 1} = \frac{32}{1} = 32.$ Ans.

(58) (*a*) $\dfrac{7}{\;\frac{3}{16}\;} = 7 \div \frac{3}{16} = 7 \times \frac{16}{3} = \frac{7 \times 16}{3} = \frac{112}{3} = 37\frac{1}{3}.$ Ans.

The heavy line indicates that 7 is to be divided by $\frac{3}{16}$.

(*b*) $\dfrac{\;\frac{15}{32}\;}{\;\frac{5}{8}\;} = \frac{15}{32} \div \frac{5}{8} = \frac{15}{32} \times \frac{8}{5} = \frac{\overset{3}{\cancel{15}} \times \cancel{8}}{\underset{4}{\cancel{32}} \times \cancel{5}} = \frac{3}{4} = .75.$ Ans.

(*c*) $\dfrac{1.25 \times 20 \times 3}{\;\frac{87 + 88}{459 + 32}\;} = ?$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

$$\begin{array}{r} 1.25 \\ \times \quad 20 \\ \hline 25.00 \\ \times \quad 3 \\ \hline 75 \end{array}$$

Multiplying the factors of the numerator together, we find their product to be 75.

The fraction $\frac{87+88}{459+32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87 + 88 = 175$.

The value of the denominator of this fraction is equal to $459 + 32 = 491$. The problem then becomes

$$\frac{75}{\frac{175}{491}} = \frac{\frac{75}{1}}{\frac{175}{491}} = \frac{75}{1} \div \frac{175}{491} = \frac{75}{1} \times \frac{491}{175} = \frac{\overset{3}{\cancel{75}} \times 491}{\underset{7}{\cancel{175}}} = \frac{1{,}473}{7} = 210\tfrac{3}{7}. \quad \text{Ans.}$$

(59) The pitch of the rivets is the distance between the centers of the rivets. Hence, since the distance around the cylindrical boiler is 166.85 in., and there are 72 rivets in one of the seams, the pitch of the rivets equals $166.85 \div 72 = 2.317 +$ in. Ans.

```
72 ) 166.850 ( 2.317+
     144
     ---
      228
      216
      ---
       125
        72
       ---
        530
        504
        ---
         26
```

(60) If a keg containing 133 boiler rivets weighs 100 pounds, then each rivet must weigh as much as 133 is contained times in 100, or .75 of a pound.

```
133 ) 100.00 ( .75+   Ans.
       931
       ---
        690
        665
        ---
         25
```

Since there are 2 decimal places in the dividend and 0 decimal places in the divisor, we must point off $2 - 0 = 2$ decimal places in the quotient, or answer.

(61) If the distance around a fly-wheel is 56.5488 ft., which is 3.1416 times its diameter, then its diameter must equal $56.5488 \div 3.1416 = 18$ ft.

```
3.1416 ) 56.5488 ( 18 ft.
         31416
         251328
         251328
              0
```

If the diameter of the fly-wheel is 18 ft., then it is evident that the diameter of a wheel one-half as large must be $\frac{1}{2}$ of 18 ft., or 9 ft.

(62) Since 1 load consists of 1,600 bricks, 8 loads consist of $8 \times 1,600 = 12,800$ bricks. Since 1 team can draw 12,800 bricks in 1 day, 6 teams can draw 6 times as many, or $6 \times 12,800 = 76,800$ bricks in 1 day. Since 6 teams can draw 76,800 bricks in 1 day, to draw 980,000 bricks, it will take them as many days as 76,800 is contained times in 980,000, or 12.76 + days. Ans.

```
76800 ) 980000.00 ( 12.76+
        76800
        212000
        153600
          584000
          537600
            464000
            460800
              3200
```

Since there are 2 decimal places in the dividend and 0 decimal places in the divisor, we must point off $2 - 0 = 2$ decimal places in the quotient.

(63) 28 acres of land at \$133 an acre would cost $28 \times \$133 = \$3,724$.

```
   28
 1064
 266
$3724
```

If a mechanic earns \$1,500 a year, and his expenses are \$968 per year, then he would save \$1 5 0 0 — \$968, or \$532 per year.

$$\begin{array}{r} \$1500 \\ 968 \\ \hline \$532 \end{array}$$

If he saves \$532 in one year, to save \$3,724 it would take as many years as \$532 is contained times in \$3,724, or 7 years.

$$\begin{array}{r} 532\,)\,3724\,(\,7 \text{ years. Ans.} \\ 3724 \\ \hline 0 \end{array}$$

(64) $\frac{7}{8}$ = value of the fraction, and 28 the numerator. We find that 4 multiplied by 7 = 28, so multiplying 8, the denominator of the fraction, by 4, we have 32 for the required denominator, and $\frac{28}{32} = \frac{7}{8}$. Hence, 32 is the required denominator.

(65) 1 plus .001 = 1.001. .01 plus .000001 = .010001. And 1.001 — .010001 =

$$\begin{array}{r} 1.001 \\ .010001 \\ \hline .990999 \end{array} \text{ Ans.}$$

(66) If the freight train ran 365 miles in one week, and 3 times as far, lacking 246 miles the next week, then it ran (3 × 365 miles) — 246 miles, or 849 miles the second week. Thus,

$$\begin{array}{r} 365 \\ 3 \\ \hline 1095 \\ -\ 246 \\ \hline 849 \end{array} \text{ miles. Ans.}$$

(67) The distance from Philadelphia to Pittsburg is 354 miles. Since there are 5,280 feet in 1 mile, in 354 miles there are 354 × 5,280 feet, or 1,869,120 feet. If the driving wheel of the locomotive is 16 feet in circumference, in going

from Philadelphia to Pittsburg, a distance of 1,869,120 feet, it will make 1,869,120 ÷ 16, or 116,820 revolutions.

```
16 ) 1869120 ( 116820 rev.   Ans.
     16
     ---
      26
      16
      ---
      109
       96
      ---
       131
       128
       ---
         32
         32
         --
          0
```

(68) $.875 = \frac{875}{1{,}000} = \frac{175}{200} = \frac{7}{8}$ of a foot.

1 foot = 12 inches.

$$\frac{7}{8} \text{ of 1 foot} = \frac{7}{\underset{2}{\cancel{8}}} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{21}{2} = 10\frac{1}{2} \text{ inches.} \quad \text{Ans.}$$

(69) 12 inches = 1 foot.

$$\frac{3}{16} \text{ of an inch} = \frac{3}{16} \div 12 = \frac{\cancel{3}}{16} \times \frac{1}{\underset{4}{\cancel{12}}} = \frac{1}{64} \text{ of a foot.}$$

```
 1
 --
 64)1.000000(.015625 Ans.
     64
     ---
     360
     320
     ---
      400
      384
      ---
       160
       128
       ---
        320
        320
        ---
          0
```

Point off 6 decimal places in the quotient, since we annexed six ciphers to the dividend, the divisor containing no decimal places; hence, 6 − 0 = 6 places to be pointed off.

(70) 6 inches $= \frac{6}{12}$ of a foot, since 12 inches $=$ 1 foot.

12 feet 6 inches $= 12\frac{6}{12} = 12\frac{1}{2}$ feet $= \frac{(12 \times 2) + 1}{2} = \frac{25}{2}$ feet.

If there are 7 pieces of pipe and each piece is $12\frac{1}{2}$ feet long, then the whole length of the pipe would be $\frac{25}{2} \times 7 = \frac{175}{2} = 87.5$ feet.

12 inches $=$ 1 foot. And $\frac{3}{4}$ inch $= \frac{3}{4} \div 12 = \frac{3}{4} \div \frac{12}{1} = \frac{\cancel{3}}{4} \times \frac{1}{\underset{4}{\cancel{12}}} = \frac{1}{16}$ of a foot to be allowed at each joint for screwing.

$\frac{1}{16}$ reduced to its equivalent decimal $= .0625$ of a foot.

The first length screws into the tank $\frac{1}{16}$ or .0625 of a foot, thereby shortening the length of the pipe .0625 of a foot. The length of the pipe now equals 87.5 feet $-$ .0625 foot $=$ 87.4375 feet.

$$\begin{array}{rl} 87.5 & \text{feet} \\ -\quad .0625 & \text{feet} \\ \hline 87.4375 & \text{feet} \end{array}$$

The $\frac{3}{4}$ of an inch, or $\frac{1}{16}$ of a foot, allowed at each of the other 6 joints must be added to the length of the pipe since the different lengths are connected by unions which prevent the ends of the pipe from coming together, and, in this case, keep them $\frac{3}{4}''$ apart. Hence, we have $6 \times \frac{1}{16} = \frac{6}{16}$ or .375 of a foot for the 6 joints.

These 6 joints lengthen the pipe .375 of a foot; consequently, the water will be discharged at a distance of 87.4375 feet $+$ .375 foot, or 87.8125 feet from the tank.

$$\begin{array}{rl} 87.4375 & \text{feet} \\ +\quad .375 & \text{feet} \\ \hline 87.8125 & \text{feet} \quad \text{Ans.} \end{array}$$

(71) 72.6 feet of fencing at \$.50 a foot would cost 72.6 × .50, or \$36.30.

$$\begin{array}{r} 72.6 \\ .50 \\ \hline \$36.300 \end{array}$$

If, by selling a carload of coal at a profit of \$1.65 per ton, I make \$36.30, then there must be as many tons of coal in the car as 1.65 is contained times in 36.30, or 22 tons.

$$\begin{array}{l} 1.65)36.30(22 \text{ tons. Ans.} \\ \quad\;\;\; 330 \\ \quad\;\;\; \overline{\;\;\;330} \\ \quad\;\;\;\;\;\; 330 \\ \quad\;\;\;\;\;\; \overline{\quad\;\; 0} \end{array}$$

(72) 14 ft. 5 in. $= 14\frac{5}{12}$ ft., since 12 inches equal 1 foot. Of the six pieces of connecting pipe between an engine and boiler, there are three pieces each measuring 14 ft. 5 in., or $14\frac{5}{12}$ ft.; hence, the total length of the three pieces equals $3 \times 14\frac{5}{12} = 3 \times \frac{173}{12} = \frac{519}{12} = 43\frac{1}{4}$ ft.

Since each of the other two pieces is 12 ft. 6 in., or $12\frac{1}{2}$ ft. in length, the total length of these two pieces equals $2 \times 12\frac{1}{2}$ ft. $= 25$ ft. The remaining piece of pipe is 8 ft. 10 in., or $8\frac{5}{6}$ ft. in length, since 10 in. $= \frac{10}{12}$, or $\frac{5}{6}$ of a ft. (there being 12 inches in 1 foot). Adding these different lengths will give the total length of the six pieces of pipe. However, before adding we will reduce the fractions of the two mixed numbers $43\frac{1}{4}$ and $8\frac{5}{6}$ to fractions having a common denominator. Doing this, we have $\frac{1}{4} = \frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$; $\frac{5}{6} = \frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$. $\frac{3}{12} + \frac{10}{12} = \frac{13}{12} = 1\frac{1}{12}$ ft. The sum of the whole

numbers equals 43 ft. + 25 ft. + 8 ft. = 76 ft., which added to $1\frac{1}{12}$ ft. gives $77\frac{1}{12}$ ft. as the total length of the connecting pipe. Since 1 ft. of pipe weighs $10\frac{1}{2}$ pounds, then it is clearly seen that this pipe would weigh $77\frac{1}{12} \times 10\frac{1}{2} = \frac{925}{\cancelto{4}{12}} \times \frac{\cancelto{7}{21}}{2} = \frac{6,475}{8} = 809\frac{3}{8}$, or 809.875 lb. Ans.

$$\begin{array}{r} 77 \\ \times\ 12 \\ \hline 154 \\ 77 \\ \hline 924 \\ +\ \ 1 \\ \hline 925 \end{array} \qquad \frac{3}{8} = .375, \text{ since } 8\,\underline{)\,3.000} \atop .375$$

(73) Since these four bolts measure $2\frac{1}{2}$, $6\frac{7}{8}$, $3\frac{1}{16}$, and 4 inches, respectively, together they will measure $16\frac{7}{16}$ inches, since $2\frac{1}{2} + 6\frac{7}{8} + 3\frac{1}{16} + 4 = 16\frac{7}{16}$. Reducing the fractions of the mixed numbers to a common denominator, we have $\frac{1}{2} = \frac{1}{2} \times \frac{8}{8} = \frac{8}{16}$; $\frac{7}{8} = \frac{7}{8} \times \frac{2}{2} = \frac{14}{16}$; $\frac{8}{16} + \frac{14}{16} + \frac{1}{16} = \frac{8+14+1}{16} = \frac{23}{16} = 1\frac{7}{16}$. $2 + 6 + 3 + 4 + 1\frac{7}{16} = 16\frac{7}{16}$. If $\frac{7}{16}$ of an inch is allowed for cutting and finishing each bolt, then the allowance for the 4 bolts would equal $4 \times \frac{7}{16} = \frac{28}{16} = 1\frac{12}{16}$ inches, which added to $16\frac{7}{16}$ inches equals $18\frac{3}{16}$ inches, the length of the piece of iron required. Ans.

$$\begin{array}{r} 16\frac{7}{16} \\ 1\frac{12}{16} \\ \hline 17\frac{19}{16} \end{array}$$

$17\frac{19}{16}$, or $18\frac{3}{16}$, since $\frac{19}{16} = 1\frac{3}{16}$.

(74) A double belt of a certain width can transmit $64\frac{1}{2}$, or 64.5 horsepower. If this is $\frac{10}{7}$ of the power transmitted by one single belt, then $\frac{1}{7}$ of the power transmitted would equal $\frac{1}{10}$ of 64.5, or 6.45 horsepower, and $\frac{7}{7}$ would equal 7×6.45 horsepower, or 45.15 horsepower. Since 1 single belt of a certain width can transmit 45.15 horsepower, it is clearly evident that 2 single belts, when running under the same conditions, would transmit 2×45.15, or 90.3 horsepower. Ans.

(75) Before solving, we will reduce the several lengths to the same denomination, in this case feet. Since 12 inches = 1 foot,

$$18 \text{ ft. } 6 \text{ in.} = 18\frac{6}{12}, \text{ or } 18\frac{1}{2} \text{ ft.};$$

$$16 \text{ ft. } 9\tfrac{1}{2} \text{ in.} = 16\frac{9\frac{1}{2}}{12} = 16\frac{\frac{19}{2}}{12} = 16\frac{19}{24};$$

$$\left(\frac{19}{2} \div 12 = \frac{19}{2} \div \frac{12}{1} = \frac{19}{2} \times \frac{1}{12} = \frac{19}{24}\right);$$

$$22 \text{ ft. } 2 \text{ in.} = 22\frac{2}{12}, \text{ or } 22\frac{1}{6} \text{ ft.};$$

$$20 \text{ ft. } 8\tfrac{1}{2} \text{ in.} = 20\frac{8\frac{1}{2}}{12} = 20\frac{\frac{17}{2}}{12} = 20\frac{17}{24}.$$

$$\left(\frac{17}{2} \div 12 = \frac{17}{2} \div \frac{12}{1} = \frac{17}{2} \times \frac{1}{12} = \frac{17}{24}\right).$$

Adding the different lengths, we have

$$18\frac{1}{2} \text{ ft.} + 16\frac{19}{24} \text{ ft.} + 22\frac{1}{6} \text{ ft.} + 20\frac{17}{24} \text{ ft.} = 78\frac{1}{6} \text{ ft.}$$

Reducing $\frac{1}{2}$, $\frac{19}{24}$, $\frac{1}{6}$, and $\frac{17}{24}$ to fractions having a common denominator, as in problem 42, we have $\frac{1}{2} = \frac{12}{24}$; $\frac{19}{24} = \frac{19}{24}$; $\frac{1}{6} = \frac{4}{24}$, and $\frac{17}{24} = \frac{17}{24}$. Adding, we have $\frac{12 + 19 + 4 + 17}{24} =$

$\frac{52}{24} = 2\frac{4}{24}$, or $2\frac{1}{6}$, which added to the sum of the whole numbers, or 76, equals $78\frac{1}{6}$ ft. Ans.

$$\begin{array}{rl} 18\frac{1}{2} & \text{ft.} \\ 16\frac{11}{12} & \text{ft.} \\ 22\frac{1}{6} & \text{ft.} \\ 20\frac{7}{12} & \text{ft.} \\ \hline 76 & \text{ft.} \\ 2\frac{1}{6} & \text{ft.} \\ \hline 78\frac{1}{6} & \text{ft.} \end{array}$$

(76) The total horsepower developed equals $48.63 + 45.7 + 46.32 + 47.9 + 48.74 + 48.38 + 48.59 = 334.26$.

Since the horsepower developed equals 334.26, then the average horsepower developed must equal $334.26 \div 7$, or $47.75 +$ H. P.

$$\begin{array}{r} 48.63 \\ 45.7 \\ 46.32 \\ 47.9 \\ 48.74 \\ 48.38 \\ 48.59 \\ \hline 334.26 \end{array}$$

ARITHMETIC.

(QUESTIONS 77–141.)

---

(77) A certain per cent. of a number means so many hundredths of that number.

25% of 8,428 lb. means 25 hundredths of 8,428 lb. $\frac{25}{100} =$ .25. Hence, 8,428 lb. $\times$ .25 $=$ 2,107 lb. Ans.

(78) 1% means one-hundredth $\left(\frac{1}{100}\right)$, which is expressed decimally as .01. Then, \$100 $\times$ .01 $=$ \$1.

$$\begin{array}{r} \$100 \\ .01 \\ \hline \$1.00 \end{array} \text{ Ans.}$$

(79) $\frac{1}{2}$% means one-half of 1 per cent. Since 1% is .01, $\frac{1}{2}$% is .005, for $2\overline{)\,.010}$ giving $.005$. And \$35,000 $\times$ .005 $=$ \$175. Ans.

$$\begin{array}{r} \$35000 \\ .005 \\ \hline \$175.000 \end{array}$$

(80) If 2 is a certain per cent. of 50, then 50 multiplied by a certain rate gives a product of 2, and that rate is equal to 2 divided by 50. Dividing 2 by 50, the quotient is .04, which means that 2 is 4% of 50, or, since percentage $=$ base $\times$ rate,

$$50\,)\,2.00\,(\,.04 \text{ Ans.}$$
$$\underline{2\,00}$$

rate $=$ percentage $\div$ base
$= 2 \div 50 = .04$, or 4%. Ans.

(81) Since percentage = base × rate, rate = percentage ÷ base.*

As percentage = 10 and base = 10, we have rate = 10 ÷ 10 = 1.

But $1 = \frac{100}{100}$ and $\frac{100}{100} = 100\%$; hence, the rate (1) means that 10 is 100% of 10.

(82) (*a*) Rate = percentage ÷ base.

As percentage = $176.54 and base = $2,522, we have rate = 176.54 ÷ 2,522 = .07 = 7%. Ans.

```
2522 ) 176.54 ( .07
       17654
```

(*b*) Base = percentage ÷ rate.

As percentage = 16.96 and rate = 8% = .08, we have base = 16.96 ÷ .08 = 212. Ans.

```
.08 ) 16.96 ( 212
      16
       9
       8
       16
       16
```

(*c*) Amount is the sum of the base and percentage; hence, the percentage = amount minus the base.

Amount = 216.7025 and base = 213.5; hence, percentage = 216.7025 − 213.5 = 3.2025.

Rate = percentage ÷ base.

As percentage = 3.2025 and base = 213.5, we have rate = 3 2025 ÷ 213.5 = .015. Ans.

```
213.5 ) 3.2025 ( .015 = 1½%
        2135
        10675
        10675
```

*Remember that an expression of this form means that the first term is to be divided by the second term. Thus, as above, it means percentage divided by base.

(*d*) The difference is the remainder found by subtracting the percentage from the base; hence, base − the difference = the percentage. Base = 207 and difference = 201.825; hence, percentage = 207 − 201.825 = 5.175.

Rate = percentage ÷ base.

As percentage = 5.175 and base = 207, we have rate = 5.175 ÷ 207 = .025 = $.02\frac{1}{2} = 2\frac{1}{2}\%$. Ans.

```
207 ) 5.175 ( .025
      414
      ----
      1035
      1035
      ----
```

(83) Since 5,500 lb. represent an increase of 15% over the consumption when the condenser is used, 5,500 lb. must be the amount, 15% the rate, and the number of pounds consumed when the condenser is running (to be found) the base.

Base = amount ÷ (1 + rate) = 5,500 ÷ (1 + .15)
= 5,500 ÷ 1.15 = 4,782.61 lb., nearly. Ans.

```
1.15 ) 5500.0000 ( 4782.61
       460
       ----
        900
        805
        ----
         950
         920
         ----
          300
          230
          ----
           700
           690
           ----
            100
            115
            ----
```

Or, this problem could also have been solved as follows:

100% = the number of pounds consumed when the condenser is running. If there is a gain of 15%, then 100% + 15%, or 115% = 5,500 lb., the amount used when the condenser is not running. If 115% = 5,500 lb., 1% = $\frac{1}{115}$ of 5,500 = 47.8261 lb., and 100% = 100 × 47.8261 = 4,782.61 lb. Ans.

(84)
$$\begin{array}{llll}
24\ \% \text{ of } \$950 = 950 \times .24 & = \$228 \\
12\frac{1}{2}\% \text{ of } \$950 = 950 \times .125 & = 118.75 \\
\underline{17}\ \% \text{ of } \$950 = 950 \times .17 & = \underline{161.50} \\
53\frac{1}{2}\% \text{ of } \$950 & = \$508.25
\end{array}$$

The total amount of his yearly expenses, then, is \$508.25; hence, his savings are \$950 − \$508.25 = \$441.75. Ans.

Or, as above, $24\% + 12\frac{1}{2}\% + 17\% = 53\frac{1}{2}\%$, the total percentage of expenditures; hence,

\$950 × .535 = \$508.25, and

\$950 − \$508.25 = \$441.75 = his yearly savings. Ans.

(85) The percentage is 961.38 and the rate is $37\frac{1}{2}\%$.

Base = percentage ÷ rate

= 961.38 ÷ .375 = 2,563.68, the number. Ans.

Another method of solving is the following:

If $37\frac{1}{2}\%$ of a number is 961.38, then $.37\frac{1}{2}$ times the number = 961.38 and the number = $961.38 \div .37\frac{1}{2}$, which, as above = 2,563.68. Ans.

```
.375 ) 961.38000 ( 2563.68
       750
       2113
       1875
        2388
        2250
         1380
         1125
          2550
          2250
           3000
           3000
```

(86) Here \$1,125 is 30% of some number; hence, \$1,125 = the percentage, 30% = the rate, and the required number is the base.

Base = percentage ÷ rate.

= \$1,125 ÷ .30 = \$3,750.

Since \$3,750 is $\frac{3}{4}$ of the property, one of the fourths is $\frac{1}{3}$

of \$3,750 = \$1,250, and $\frac{4}{4}$, or the entire property, is 4 × \$1,250 = \$5,000. Ans.

(87) Here, \$4,810 is the difference and 35% the rate.
Base = difference ÷ (1 − rate)
= \$4,810 ÷ (1 − .35) = \$4,810 ÷ .65 = \$7,400. Ans.

```
.65 ) 4810.00 ( 7400
      455
      ---
       260                    1.00
       260                     .35
       ---                     ---
                               .65
```

The solution can also be effected as follows: 100% = the sum diminished by 35%, then (1 − .35) = .65, which is \$4,810.

If 65% = \$4,810, 1% = $\frac{1}{65}$ of 4,810 = \$74, and 100% = 100 × \$74 = \$7,400. Ans.

(88) The volume of the clearance in a steam-engine cylinder = 18.3 cu. in., and the volume of the cylinder, neglecting the clearance, = 254.5 cu. in. We wish to know what percentage of the cylinder volume is the clearance, or what per cent. of 254.5 is 18.3.

254.5 is the base.

18.3 is the percentage.

The rate, which we wish to find, equals the percentage ÷ base = 18.3 ÷ 254.5 = 7.2% nearly.

```
254.5 ) 18.3000 ( .072, or 7.2%, nearly.
        17815
        -----
          4850
          5090
          ----
```

(89) 16.5 miles = $12\frac{1}{2}$% of the entire length of the road

We wish to find the *entire* length.

16.5 miles is the percentage and $12\frac{1}{2}$% is the rate, and the entire length will be the base.

$$\text{Base} = \text{percentage} \div \text{rate}$$
$$= 16.5 \div .12\tfrac{1}{2}.$$

```
.125)16.500(132 miles.   Ans.
     125
     ---
      400
      375
      ---
       250
       250
       ---
```

(90) 298 revolutions per minute with the load = base. $1\frac{1}{2}\%$ = rate, and the amount (to be found) will equal the speed of the engine when running unloaded.

Amount = base × (1 + rate)

$= 298 \times (1 + .015) = 302.47$ rev. per min. Ans.

```
      298
  ×  1.015
     -----
     1490
     298
    000
   298
   -------
   302.470
```

(91) 4 yd. 2 ft. 10 in. to inches.

```
 ×   3
    --
    12
 +   2
    --
    14 feet
 ×  12
    --
    28
   14
   ---
   168
 +  10
   ---
   178 inches.  Ans.
```

Since there are 3 feet in one yard, in 4 yards there are 4 × 3 feet, or 12 feet. 12 feet plus 2 feet = 14 feet.

There are 12 inches in one foot; therefore, in 14 feet there are 12 × 14 or 168 inches. 168 inches plus 10 inches = 178 inches. Ans.

(92)

$$\begin{array}{r l} 12\,)\,\underline{3722} & \text{inches.} \\ 3\,)\,\underline{310} & +2 \text{ inches.} \\ 103 & +1 \text{ foot.} \end{array}$$

Ans. = 103 yd. 1 ft. 2 in.

EXPLANATION.—There are 12 inches in one foot; hence, in 3,722 inches there are as many feet as 12 is contained times in 3,722, or 310 feet and 2 inches remaining. Write 2 inches as a remainder. There are 3 feet in one yard; hence, in 310 yards there are as many feet as 3 is contained times in 310, or 103 yards and 1 foot remaining. Hence, in 3,722 inches there are 103 yd. 1 ft. 2 in.

(93)

$$\begin{array}{r l} & 5 \text{ weeks } 3.5 \text{ days.} \\ \times & \underline{7} \\ & 35 \text{ days in 5 weeks.} \\ + & \underline{3.5} \\ & 38.5 \text{ days.} \end{array}$$

Then, we find how many seconds there are in 38.5 days.

$$\begin{array}{r l} 38.5 & \text{days.} \\ \times\ \underline{24} & \text{hours in one day.} \\ 1540 & \\ \underline{770} & \\ 924.0 & \text{hours in 38.5 days.} \\ \times\ \underline{60} & \text{minutes in one hour.} \\ 55440 & \text{minutes in 38.5 days.} \\ \times\ \underline{60} & \text{seconds in one minute.} \\ 3326400 & \text{seconds in 38.5 days. Ans.} \end{array}$$

(94)

$$\begin{array}{r l} 1728\,)\,\underline{764325} & \text{cu. in.} \\ 27\,)\,\underline{442} & +549 \text{ cu. in.} \\ 16 & \text{cu. yd.} + 10 \text{ cu. ft.} \end{array}$$

Ans. = 16 cu. yd. 10 cu. ft. 549 cu. in.

EXPLANATION.—There are 1,728 cubic inches in one cubic foot; hence, in 764,325 cu. in. there are as many cubic feet as 1,728 is contained times in 764,325, or 442 cubic feet, and 549 cubic inches remaining. Write the 549 cubic inches as a remainder. There are 27 cubic feet in one cubic yard; hence, in 442 cubic feet there are as many cubic yards as 27

is contained times in 442 cubic feet, or 16 cubic yards and 10 cubic feet remaining. Then, in 764,325 cubic inches there are 16 cu. yd. 10 cu. ft. 549 cu. in. Ans.

(95) There are $31\frac{1}{2}$ (31.5, since $\frac{1}{2} = .5$) gallons in 1 barrel; hence, in 4 barrels there are 4 times 31.5, or 126 gallons. 126 gallons plus 10 gallons = 136 gallons.

$$\begin{array}{rlll} & \text{bbl.} & \text{gal.} & \text{qt.} \\ & 4 & 10 & 3 \\ \times & 31.5 & & \\ \hline & 126.0 & & \\ + & 10 & & \\ \hline & 136 & \text{gallons.} & \\ \times & 4 & & \\ \hline & 544 & & \\ + & 3 & & \\ \hline 4) & 547 & \text{quarts.} & \\ \hline & 136\frac{3}{4} & \text{gallons. Ans.} & \end{array}$$

There are 4 quarts in 1 gallon; hence, in 136 gallons there are 136 times 4 = 544 quarts.

544 quarts plus 3 quarts = 547 quarts. Since the tank under consideration contains 547 quarts, and since 4 quarts = 1 gallon, the tank would contain as many gallons of water as 4 is contained times in 547, or $136\frac{3}{4}$ gallons. Ans.

(96)

$$\begin{array}{rlll} & \text{T.} & \text{cwt.} & \text{lb.} \\ & 16 & 8 & 75 \\ \times & 20 & & \\ \hline & 320 & & \\ + & 8 & & \\ \hline & 328 & \text{cwt.} & \\ \times & 100 & & \\ \hline & 32800 & & \\ + & 75 & & \\ \hline & 32875 & \text{lb.} & \end{array}$$

Since in one ton there are 20 cwt., in 16 tons there are $16 \times 20 = 320$ cwt. 320 cwt. + 8 cwt. = 328 cwt. There are 100 lb. in 1 cwt.; hence, in 328 cwt. there are $328 \times 100 = 32,800$ lb. 32,800 lb. + 75 lb. = 32,875 lb. Ans.

(97)

$$\begin{array}{r} 100)\underline{25396} \text{ lb.} \\ 20)\underline{253} \text{ cwt.} + 96 \text{ lb.} \\ 12 \text{ T.} + 13 \text{ cwt.} \end{array}$$

There are 100 lb. in 1 cwt.; hence, in 25,396 lb. there are as many cwt. as 100 is contained times in 25,396, or 253 cwt. and 96 lb. remaining.

There are 20 cwt. in 1 ton, and in 253 cwt. there are as many tons as 20 is contained times in 253, or 12 tons and

13 cwt. remaining. Hence, 25,396 lb. = 12 T. 13 cwt. 96 lb. Ans.

(98) There are 2 pt. in 1 qt.; hence, in 25,396 pt. there are 12,698 qt. There are 4 qt. in one gal.; hence, in 12,698 qt. there are as many gal. as 4 is contained times in 12,698, or 3,174 gal. with 2 qt. remaining.

$$\begin{array}{r|l} 2\,) & \underline{25396} \text{ pt.} \\ 4\,) & \underline{12698} \text{ qt.} \\ 31.5\,) & \underline{3174} \text{ gal.} + 2 \text{ qt.} \\ & \quad 100 \text{ bbl.} + 24 \text{ gal.} \end{array}$$

Since $31\frac{1}{2}$, or 31.5 gal. make 1 bbl., in 3,174 gal. there would be as many bbl. as 31.5 is contained times in 3,174, or 100 bbl. and 24 gal. remaining.

$$\begin{array}{r} 31.5\,)\,3174.0 \text{ gal.}\,(\,100 \text{ bbl.} \\ \underline{315} \\ 24.0 \end{array}$$

Hence, 25,396 pt. = 100 bbl. 24 gal. 2 qt. Ans.

(99) Arrange the different terms in columns, taking care to have like denominations in the same column. We begin to add at the right-hand column. $7 + 9 + 3 = 19$ in.; since 12 in. = 1 ft., 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column, and reserve the 1 ft. to add to the sum of the feet. $2 + 1 + 2 + 1$ (reserved) = 6 ft. Since 3 ft. = 1 yd., 6 ft. = 2 yd. and 0 ft. remaining. Place the 0 in the feet column and reserve the 2 yd. to add to the sum of the yards. $4 + 2 + 2$ (reserved) = 8 yd., which we place in yards column. Ans. = 8 yd. 7 in.

yd.	ft.	in.	
2	2	3	
4	1	9	
	2	7	
8	0	7	Ans.

(100) Since "pints" is the lowest denomination, we will find their sum first. $5 + 1 + 1 = 7$ pt. Since there are 2 pt. in 1 qt., in 7 pt. there are as many quarts as 2 is contained in 7, or 3 qt. and 1 pt. remaining. Place 1 pt. under pints column and add

gal.	qt.	pt.	
3	3	1	
6	0	1	
4	3	5	
16	2	1	Ans.

3 qt. to the sum of the quarts. $8 + 3 + 3$ (reserved) $= 14$ qt. Since there are 4 qt. in 1 gal., in 14 qt. there are as many gallons as 4 is contained in 14, or 3 gal., and 2 qt. remaining. Place 2 qt. under quarts column and reserve the 3 gal. to add to the sum of the gallons. $4 + 6 + 3 + 3$ (reserved) $=$ 16 gal. Hence, the answer is 16 gal. 2 qt. 1 pt.

(101) We will first reduce 115 ft. to yards. Since 3 ft. = 1 yd., 115 ft. = 38 yd. and 1 ft. Place the subtrahend under the minuend, so that like denominations are under each other. Since the inch is the lowest denomination, we subtract the inches first.

yd.	ft.	in.	
52	2	9	
38	1	0	
14	1	9	Ans.

0 in. subtracted from 9 in. = 9 in., which we write in inches place in the remainder. 1 ft. subtracted from 2 ft. = 1 ft., which we write in the remainder. 38 yd. subtracted from 52 yd. = 14 yd., which we write in yards place in the remainder. Hence, the remainder, or answer = 14 yd. 1 ft. 9 in.

(102) Since 10 gal. 2 qt. 1 pt. of machine oil is sold at one time, and 16 gal. 3 qt. at another time, together there was sold 27 gal. 1 qt. 1 pt. $0 + 1 = 1$ pt. We can not reduce 1 pt. to any higher denomination, so place it under pints column. 3 qt. + 2 qt. = 5 qt. Since 4 qt. = 1 gal., 5 qt. = 1 gal., and 1 qt. remaining. Place 1 qt. under quarts column and reserve the 1 gal. to add to the gallons. 16 gal. + 10 gal. + 1 gal. (reserved) = 27 gal. Since the barrel contained $31\frac{1}{2}$, or 31.5 gal., and 27 gal. 1 qt. 1 pt. were sold, there remained the difference, or 4 gal. 1 pt. 31.5 gal. = 31 gal. 2 qt., since $.5 = \frac{1}{2}$, and $\frac{1}{2}$ of 1 gal. = $\frac{1}{2}$ of 4 qt. = 2 qt. 1 pt. can not be taken from 0 pt., so we take 1 qt. from the 2 qt. The 1 qt. taken = 2 pt. 1 pt. from 2

gal.	qt.	pt.
10	2	1
16	3	0
27	1	1

gal.	qt.	pt.	
31	2	0	
27	1	1	
4	0	1	Ans.

pt. = 1 pt. Place 1 pt. under pints column. Since we took 1 qt. from the quarts column, there remains 2 − 1, or 1 qt. 1 qt. from 1 qt. leaves 0 qt. Place 0 qt. under the quarts column. 27 gal. from 31 gal. leaves 4 gal. Place 4 gal. under the gallons column. We, therefore, find that 4 gal. 1 pt. of machine oil remained in the barrel.

(103) In multiplication of denominate numbers, we place the multiplier under the lowest denomination of the multiplicand, as

$$\begin{array}{r} 17 \text{ ft. } 3 \text{ in.} \\ 51 \\ \hline 879 \text{ ft. } 9 \text{ in.} \end{array}$$

and begin at the right to multiply. $51 \times 3 = 153$ in. Since there are 12 in. in 1 ft., in 153 in. there are as many feet as 12 is contained times in 153, or 12 ft. and 9 in. remaining. Place the 9 in. under the inches and reserve the 12 ft. 51×17 ft. = 867 ft.; 867 ft. + 12 ft. (reserved) = 879 ft. 879 ft. can be reduced to higher denominations by dividing by 3 ft. to find the number of yards, and by $5\frac{1}{2}$ to find the number of rods.

$$\begin{array}{r} 3\underline{)879} \text{ ft. } 9 \text{ in.} \\ 5.5\underline{)293} \text{ yd.} \\ 53 \text{ rd. } 1\frac{1}{2} \text{ yd.} \end{array}$$

Then, Ans. = 53 rd. $1\frac{1}{2}$ yd. 0 ft. 9 in., or 53 rd. 1 yd. 2 ft. 3 in.

(104) Since 2 pt. = 1 qt., 3 qt. = 3×2, or 6 pt. 6 pt. + 1 pt. = 7 pt. $4.7 \times 7 = 32.9$ pt. Ans.

$$\begin{array}{rr} \text{qt.} & \text{pt.} \\ 3 & 1 \\ \times\ 2 & \\ \hline 6 & \\ +\ 1 & \\ \hline 7 \text{ pt.} & \end{array} \qquad \begin{array}{r} 7 \text{ pt.} \\ 4.7 \\ \hline 49 \\ 28 \\ \hline 32.9 \text{ pt.} \end{array}$$

(105) We must first reduce 23 miles to feet before we can divide by 30 feet.

$$1 \text{ mi.} = 320 \text{ rd.}$$
$$23 \text{ mi.} = 23 \times 320 = 7{,}360 \text{ rd.}$$
$$1 \text{ rd.} = 16\frac{1}{2} \text{ or } \frac{33}{2} \text{ ft.}$$
$$7{,}360 \text{ rd.} = \overset{3680}{\cancel{7360}} \times \frac{33}{\cancel{2}} = 121{,}440 \text{ ft.}$$

121,440 ft. ÷ 30 = 4,048 rails for one side of the track.

The number of rails for 2 sides of the track = 2 × 4,048, or 8,096 rails. Ans.

(106) 15 ft. 5 in. × 4 = 60 ft. 20 in., the length of the four pieces.

ft.	in.
60	20
14	8
8	10
82	38, or 85 ft. 2 in. = the length of the shaft.

15 ft.	5 in.
	4
60 ft.	20 in.

From the length of the shaft we must subtract 8 in. × 2 = 16 in. to get the distance between the end hangers.

ft.	in.
82	38
	16
82	22, or 83 ft. 10 in.

Since there are six hangers there are *five* spaces; the length of one space is 83 ft. 10 in. ÷ 5 = 16 ft. $9\frac{1}{5}$ in. Ans.

(107) Since the distance around a wheel is $\frac{22}{7}$ times the distance across it, the distance around the wheel will equal (9 ft. $6\frac{1}{2}$ in.) $\times \frac{22}{7}$. We will reduce 9 ft. to inches before multiplying by the fraction. 9 ft. × 12 = 108 in. 108 in. + $6\frac{1}{2}$ in. = $114\frac{1}{2}$ in.

Reducing $114\frac{1}{2}$ to an improper fraction, we have $\frac{229}{2}$.

Multiplying, $\frac{229}{2} \times \frac{\overset{11}{\cancel{22}}}{7} = \frac{2,519}{7} = 359\frac{6}{7}$ in.

$$\begin{array}{r} 229 \\ \times\ 11 \\ \hline 229 \\ 229 \\ \hline 2519 \end{array}$$

Dividing $359\frac{6}{7}$ in. by 12 to reduce to feet, we have 29 ft. $11\frac{6}{7}$ in.

$$\begin{array}{l} 12)\,359\tfrac{6}{7}\text{ in. }(29\text{ ft.} \\ 24 \\ \hline 119 \\ 108 \\ \hline 11\tfrac{6}{7}\text{ in.} \end{array}$$

(108) Reducing 18 ft. $11\frac{1}{4}$ in. to inches, we have $227\frac{1}{4}$ in., or 227.25 in.

$$\begin{array}{rl} \text{ft.} & \text{in.} \\ 18 & 11\tfrac{1}{4} \\ 12 & \\ \hline 36 & \\ 18 & \\ \hline 216 & \\ 11\tfrac{1}{4} & \\ \hline 227\tfrac{1}{4}\text{ in.} & \end{array}$$

$1\frac{1}{8} \times 2$, or $2\frac{1}{4}$ in., for the two end rivets is deducted from the length, leaving 225 in., which is divided into equal spaces by the rivets.

$$\begin{array}{r} 227.25\text{ in.} \\ -\ \ 2.25\text{ in.} \\ \hline 225\text{ in.} \end{array}$$

The pitch of the rivets (or the distance between their centers) is $1\frac{1}{4}$ in., or 1.25 in.; hence, $225 \div 1.25 = 180$ spaces between the rivets. But, since there will be 1 more rivet than the number of spaces, the number of rivets required for this boiler shell will be $180 + 1 = 181$. Ans.

```
1.25)225.00(180
     125
     ----
     1000
     1000
     ----
        0
```

(109) (*a*) To find the second power of a number, we must multiply the number by itself once; that is, use the number twice as a factor. Thus, the second power of 108 is $108 \times 108 = 11{,}664$. Ans.

```
  108
  108
 ----
  864
108
-----
11664 Ans.
```

(*b*) The third power of 181.25 equals the number obtained by using 181.25 as a factor three times. Thus, the third power of 181.25 is $181.25 \times 181.25 \times 181.25 = 5{,}954{,}345.703125$. Ans.

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the first product.

Since there are 4 decimal places in the multiplicand, and 2 in the multiplier, there are $4 + 2 = 6$ decimal places in the final product.

```
          181.25
          181.25
          ------
          90625
         36250
        18125
     145000
     18125
     ----------
     32851.5625
         181.25
     ----------
    1642578125
   657031250
  328515625
2628125000
328515625
-------------
5954345703125 Ans.
```

```
          27.61
          27.61
          2761
       16566
      19327
      5522
      762.3121
          27.61
      7623121
   45738726
  53361847
 15246242
 21047.437081
          27.61
 21047437081
126284622486
147332059567
42094874162
581119.73780641
                Ans.
```

(*c*) The fourth power of 27.61 is the number obtained by using 27.61 as a factor four times. Thus, the fourth power of 27.61 is $27.61 \times 27.61 \times 27.61 \times 27.61 = 581{,}119.73780641$. Ans.

Since there are 2 decimal places in the multiplier, and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the first product.

Since there are 4 decimal places in the multiplicand, and 2 in the multiplier, there are $4 + 2 = 6$ decimal places in the second product.

Since there are 6 decimal places in the multiplicand, and 2 in the multiplier, there are $6 + 2 = 8$ decimal places in the final product.

(110) (*a*) $106^2 = 106 \times 106 = 11{,}236$. Ans.

```
  106
  106
  636
106
11236  Ans.
```

(*b*) $\left(182\frac{1}{8}\right)^2 = 182\frac{1}{8} \times 182\frac{1}{8} = 33{,}169.515625$. Ans.

```
1/8 = 8 ) 1.000 ( .125
          8
          20
          16
           40
           40
```

```
     182.125
     182.125
     910625
    364250
   182125
  364250
 1457000
182125
33169.515625  Ans.
```

Since there are 3 decimal places in the multiplier and 3 in the multiplicand, there are $3 + 3 = 6$ decimal places in the product.

(*c*) $.005^2 = .005 \times .005 = .000025$. Ans.

$$\begin{array}{r} .005 \\ .005 \\ \hline .000025 \end{array} \text{ Ans.}$$

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3 + 3 = 6$ decimal places in the product.

(*d*) $.0063^2 = .0063 \times .0063 = .00003969$. Ans.

$$\begin{array}{r} .0063 \\ .0063 \\ \hline 189 \\ 378 \\ \hline .00003969 \end{array} \text{ Ans.}$$

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4 + 4 = 8$ decimal places in the product.

(*e*) $10.06^2 = 10.06 \times 10.06 = 101.2036$. Ans.

$$\begin{array}{r} 10.06 \\ 10.06 \\ \hline 6036 \\ 1006 \\ \hline 101.2036 \end{array} \text{ Ans.}$$

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, there are $2 + 2 = 4$ decimal places in the product.

(*f*) $67.85^2 = 67.85 \times 67.85 = 4,603.6225$. Ans.

$$\begin{array}{r} 67.85 \\ 67.85 \\ \hline 33925 \\ 54280 \\ 47495 \\ 40710 \\ \hline 4603.6225 \end{array} \text{ Ans.}$$

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the product.

(*g*) $967,845^2 = 967,845 \times 967,845 = 936,723,944,025$. Ans.

$$\begin{array}{r} 967845 \\ 967845 \\ \hline 4839225 \\ 3871380 \\ 7742760 \\ 6774915 \\ 5807070 \\ 8710605 \\ \hline 936723944025 \end{array} \text{ Ans.}$$

(*h*) A fraction may be raised to any power by raising both numerator and denominator to the required power.

Thus, $\left(\frac{7}{16}\right)^2 = \frac{7}{16} \times \frac{7}{16} = \frac{7 \times 7}{16 \times 16} = \frac{49}{256}$. Ans.

(*i*) $\left(\frac{1}{4}\right)^2 = \frac{1}{4} \times \frac{1}{4} = \frac{1 \times 1}{4 \times 4} = \frac{1}{16}$. Ans.

(111) (*a*) $753^3 = 753 \times 753 \times 753 = 426{,}957{,}777$. Ans.

```
        753
        753
       ----
       2259
      3765
     5271
     ------
     567009
        753
    -------
    1701027
   2835045
  3969063
  ---------
  426957777  Ans.
```

(*b*) $987.4^3 = 987.4 \times 987.4 \times 987.4 = 962{,}674{,}279.624$. Ans.

```
       987.4
       987.4
       -----
       39496
      69118
     78992
    88866
    --------
    974958.76
        987.4
   ----------
    389983504
   682471132
  779967008
 877462884
 -------------
 962674279.624  Ans.
```

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, there are $1 + 1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the multiplicand and 1 in the multiplier, there are $2 + 1 = 3$ decimal places in the final product.

(*c*) $.005^3 = .005 \times .005 \times .005 = .000000125$. Ans.

$$\begin{array}{r} .005 \\ .005 \\ \hline .000025 \\ .005 \\ \hline .000000125 \end{array} \text{ Ans.}$$

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3 + 3 = 6$ decimal places in the first product; but, as there are only 2 figures in the product, we prefix four ciphers to make the necessary 6 decimal places.

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, there are $6 + 3 = 9$ decimal places in the final product. In this case we prefix six ciphers to form the necessary 9 decimal places.

(*d*) $.4044^3 = .4044 \times .4044 \times .4044 = .066135317184$. Ans.

$$\begin{array}{r} .4044 \\ .4044 \\ \hline 16176 \\ 16176 \\ 16176 \\ \hline .16353936 \\ .4044 \\ \hline 65415744 \\ 65415744 \\ 65415744 \\ \hline .066135317184 \end{array} \text{ Ans.}$$

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4 + 4 = 8$ decimal places in the first product.

Since there are 8 decimal places in the second multiplicand and 4 in the multiplier, there are $8 + 4 = 12$ decimal places in the final product; but, as there are only 11 figures in the product, we prefix 1 cipher to make the necessary 12 decimal places.

(*e*) $.0133^3 = .0133 \times .0133 \times .0133 = .000002352637$. Ans.

$$\begin{array}{r} .0133 \\ .0133 \\ \hline 399 \\ 399 \\ 133 \\ \hline .00017689 \\ .0133 \\ \hline 53067 \\ 53067 \\ 17689 \\ \hline .000002352637 \end{array}$$ Ans.

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, we should point off $4 + 4 = 8$ decimal places in the product; but, as there are only 5 figures in the product, we prefix three ciphers to form the eight necessary decimal places in the first product.

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we should point off $8 + 4 = 12$ decimal places in the product; but, as there are only 7 figures in the product, we prefix 5 ciphers to make the 12 necessary decimal places in the final product.

(*f*) $301.011^3 = 301.011 \times 301.011 \times 301.011 =$ $27{,}273{,}890.942264331$. Ans.

$$\begin{array}{r} 301.011 \\ 301.011 \\ \hline 301011 \\ 301011 \\ 301011 \\ 903033 \\ \hline 90607.622121 \\ 301.011 \\ \hline 90607622121 \\ 90607622121 \\ 90607622121 \\ 271822866363 \\ \hline 27273890.942264331 \end{array}$$ Ans.

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, we should point off $3 + 3 = 6$ decimal places in the first product.

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, we should point off $6 + 3 = 9$ decimal places in the final product.

(g) $\left(\frac{1}{8}\right)^3 = \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} = \frac{1 \times 1 \times 1}{8 \times 8 \times 8} = \frac{1}{512}$. Ans.

$$\begin{array}{r} 8 \\ \times\ 8 \\ \hline 64 \\ \times\ 8 \\ \hline 512 \end{array}$$

(h) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer and multiply the denominators together for the denominator of the answer.

$$\left(3\frac{3}{4}\right)^3 = \frac{15}{4} \times \frac{15}{4} \times \frac{15}{4} = \frac{15 \times 15 \times 15}{4 \times 4 \times 4} = \frac{3,375}{64} = 52.734375.$$

Ans

$$3\frac{3}{4} = \frac{(3 \times 4) + 3}{4} = \frac{12 + 3}{4} = \frac{15}{4}.$$

$$\begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline 225 \\ 15 \\ \hline 1125 \\ 225 \\ \hline 3375 \end{array}$$

$$\begin{array}{l} 64\,)\,3375.000000\,(\,52.734375 \\ \quad\ \ 320 \\ \quad\ \ \overline{175} \\ \quad\ \ 128 \\ \quad\ \ \overline{470} \\ \quad\ \ 448 \\ \quad\ \ \overline{220} \\ \quad\ \ 192 \\ \quad\ \ \overline{280} \\ \quad\ \ 256 \\ \quad\ \ \overline{240} \\ \quad\ \ 192 \\ \quad\ \ \overline{480} \\ \quad\ \ 448 \\ \quad\ \ \overline{320} \\ \quad\ \ 320 \end{array}$$

Since six ciphers were annexed to the dividend, six decimal places must be pointed off in the quotient.

(112) $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$. Ans

$$\begin{array}{r} 2 \\ 2 \\ \hline 4 \\ 2 \\ \hline 8 \\ 2 \\ \hline 16 \\ 2 \\ \hline 32 \end{array}$$ Ans.

(113) $3^4 = 3 \times 3 \times 3 \times 3 = 81$. Ans.

$$\begin{array}{r} 3 \\ 3 \\ \hline 9 \\ 3 \\ \hline 27 \\ 3 \\ \hline 81 \end{array}$$ Ans.

(114) $7^9 = 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 =$ 40,353,607. Ans.

$$\begin{array}{r} 7 \\ 7 \\ \hline 49 \\ 7 \\ \hline 343 \\ 7 \\ \hline 2401 \\ 7 \\ \hline 16807 \\ 7 \\ \hline 117649 \\ 7 \\ \hline 823543 \\ 7 \\ \hline 5764801 \\ 7 \\ \hline 40353607 \end{array}$$ Ans.

(115) (*a*) $1.2^4 = 1.2 \times 1.2 \times 1.2 \times 1.2 = 2.0736$. **Ans.**

```
     1.2
     1.2
    ----
     2 4
   1 2
   -----
   1.4 4
     1.2
   -----
   2 8 8
 1 4 4
 -------
 1.7 2 8
     1.2
 -------
 3 4 5 6
1 7 2 8
--------
2.0 7 3 6
```

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we should point off $1 + 1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we should point off $2 + 1 = 3$ decimal places in the second product.

Since there are 3 decimal places in the multiplicand and 1 in the multiplier, we should point off $3 + 1 = 4$ decimal places in the final product.

(*b*) $11^5 = 11 \times 11 \times 11 \times 11 \times 11 = 161{,}051$. **Ans.**

```
       11
       11
       --
       11
      11
      ---
      121
       11
      ---
      121
     121
     ----
     1331
       11
     ----
     1331
    1331
    -----
    14641
       11
    -----
    14641
   14641
   ------
   161051
```

(*c*) $1^6 = 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$ Ans.

(*d*) $.01^4 = .01 \times .01 \times .01 \times .01 = .00000001$. Ans.

$$\begin{array}{r} .01 \\ .01 \\ \hline .0001 \\ .01 \\ \hline .000001 \\ .01 \\ \hline .00000001 \end{array}$$

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, we should point off $2 + 2 = 4$ decimal places in the first product; but, as there is only 1 figure in the product, we prefix 3 ciphers to make the necessary 4 decimal places.

Since there are 4 decimal places in the second multiplicand and 2 in the multiplier, we should point off $4 + 2 = 6$ decimal places in the second product. It is necessary to prefix 5 ciphers to make six decimal places.

Since there are 6 decimal places in the third multiplicand and 2 in the multiplier, we should point off $6 + 2 = 8$ decimal places in the product. It is necessary to prefix 7 ciphers to make 8 decimal places in the final product.

(*e*) $.1^5 = .1 \times .1 \times .1 \times .1 \times .1 = .00001$. Ans.

$$\begin{array}{r} .1 \\ .1 \\ \hline .01 \\ .1 \\ \hline .001 \\ .1 \\ \hline .0001 \\ .1 \\ \hline .00001 \end{array}$$

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we should point off $1 + 1 = 2$ decimal places in the first product. It is necessary to prefix 1 cipher to the product.

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we should point off $2 + 1 = 3$ decimal places in the second product. It is necessary to prefix 2 ciphers to the second product.

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we should point off $3 + 1 = 4$ decimal places in the third product. We prefix three ciphers to this product.

Since there are 4 decimal places in the fourth multiplicand and 1 in the multiplier, we should point off $4 + 1$ or 5 decimal places in the final product. It is necessary to prefix 4 ciphers to this product.

(116) Evolution is the reverse of involution. In involution we find the *power* of a number by multiplying the number by itself one or more times, while in evolution we find the *number* or *root* which was multiplied by itself one or more times to make the power.

(117) (*a*)

(*a*)	1		$\sqrt{3'48'67'84.40'10} = 1867.29+$ Ans.
	1	(*b*)	1
(*d*)	20	(*c*)	248
	8		224
	28	(*e*)	2467
	8		2196
	360		27184
	6		26089
	366		109540
	6		74684
	3720		3485610
	7		3361041
	3727		124569
	7		
	37340		
	2		
	37342		
	2		
	373440		
	9		
	373449		

EXPLANATION.—(1) Divide the power into periods. In the above case, where the power consists of a whole number and a decimal, we begin at the decimal point and proceed to the *left* in pointing off the *whole* number, and towards the *right* in pointing off the *decimal*, annexing a 0 to complete the last decimal period. In *square root two* figures constitute a period.

Find the greatest single number whose square is *less* than or *equal* to 3, the first period. This is evidently 1, since $2^2 = 4$, which is greater than 3. Write 1 as the first figure of the root; also, write it to the left, as shown at (*a*). Now, multiply the 1 at (*a*) by the 1 in the root, and write the result under the first period, or 3, as shown at (*b*). Sub-

tract, and bring down the next period 48, and annex it to the remainder 2, thus making 248 the *dividend*, as shown at (*c*). Add the root already found to the 1 at (*a*), thereby obtaining 2, and annex a cipher to this 2, thus making it 20, which we call the *trial divisor*.

Divide the dividend 248 at (*c*) by the trial divisor 20 at (*d*) and obtain 8, which is probably the next figure of the root. Write 8 in the root, as shown, and also add it to 20, the trial divisor, making it 28. This is called the *complete divisor*.

(2) Multiply the complete divisor 28 by 8, the second figure of the root, and subtract the result from the dividend (*c*). The remainder is 24, to which annex the next period making 2,467, as shown at (*e*), which call the *new dividend*.

Add the second figure of the root, or 8, to the divisor 28, and annex a cipher, thus obtaining 360. Dividing 2,467 by 360, we find 6 to be the next figure of the root. Adding this last figure of the root, or 6, to 360, we get 366, which, multiplied by this last figure of the root, or 6, gives 2,196, which we write under 2,467 and subtract.

(3) Annexing the next period 84 to the remainder 271 gives 27,184 as the next new dividend. Now, adding the third figure of the root to 366, and annexing a cipher, as before, we have 3,720. Dividing 27,184 by 3,720, the result is 7, which write as the next figure of the root. Adding the fourth figure of the root, or 7, to 3,720, we get 3,727, which, multiplied by 7 of the root, gives 26,089, which write under 27,184 and subtract, obtaining 1,095 as a remainder.

(4) Annexing the next period, or 40, to the remainder 1,095 gives 109,540 as the next new dividend. Adding the last figure of the root to 3,727, and annexing a cipher, as before, the result is 37,340. Dividing 109,540 by 37,340, the result is 2, which write as the next figure of the root. Adding the fifth figure of the root, or 2, to 37,340, we obtain 37,342, which, multiplied by 2 of the root, gives 74,684, which write under 109,540 and subtract, obtaining 34,856 as a remainder.

(5) Annexing the next period, or 10, to the remainder

34,856 gives 3,485,610 as the next new dividend. Now, adding the last figure of the root to 37,342, and annexing a cipher, as before, the result is 373,440. Dividing 3,485,610 by 373,440 gives 9 as a result, which write as the next, or last figure, of the root. Adding the last figure of the root, or 9, to 373,440, we get 373,449, which, multiplied by 9 of the root, gives 3,361,041, which write under 3,485,610 and subtract. Since there is a remainder, we know the given power is not a perfect square, so we place + after the root.

In this problem there are *six* periods—four in the whole number and two in the decimal—hence, there will be *six* figures in the root (since we obtain one figure of the root for each period), four figures constituting the whole number and two figures the decimal of the root. Hence,

$$\sqrt{3,486,784.4010} = 1,867.29+.$$

(*b*) $\sqrt{9'00'00'99.40'09'00} = 3000.016+$ Ans.

(*a*)	3		
	3	(*b*)	9
(*d*)	60	(*c*)	0000994009
	0		600001
	600		39400800
	0		36000156
	6000		3400644
	0		
	60000		
	0		
	600000		
	1		
	600001		
	1		
	6000020		
	6		
	6000026		

EXPLANATION.—Beginning at the decimal point, we point off the whole number into periods of *two* figures each, proceeding from *right* to *left*; also, point off the decimals into periods of *two* figures each, proceeding from *left* to *right*. The largest number whose square is contained in the first period, 9, is 3; hence, 3 is the first figure of the root. Place 3 at the left, as shown at (*a*), and multiply it by the first

figure in the root, or 3. The result is 9. Write 9 under the first period, 9, as at (*b*), subtract, and there is no remainder. Bring down the next period, which is 00, as shown at (*c*). Add the root already found to the 3 at (*a*), obtaining 6, and annex a cipher to this 6, thus making it 60, which is the *trial divisor*, as shown at (*d*). Divide the dividend (*c*) by the trial divisor and obtain 0 as the next figure in the root. Write 0 in the root as shown, and also add it to the trial divisor 60, and annex a cipher, thereby making the next trial divisor 600. Bring down the next period, 00, annex it to the dividend already obtained and divide it by the trial divisor. 600 is contained in 0000, 0 times, so we place another cipher in the root. Write 0 in the root, as shown, and also add it to the trial divisor 600, and annex a cipher, thereby making the next trial divisor 6,000. Bring down the next period, 99. The trial divisor 6,000 is contained in 000099, 0 times, so we place 0 as the next figure in the root, as shown, and also add it to the trial divisor 6,000, and annex a cipher, thereby making the next trial divisor 60,000. Bring down the next period, 40, and annex it to the dividend already obtained to form the new dividend, 00009940, and divide it by the trial divisor 60,000. 60,000 is contained in 00009940, 0 times, so we place another cipher in the root, as shown, and also add it to the trial divisor 60,000, and annex one cipher, thereby making the next trial divisor 600,000. Bring down the next period, 09, and annex it to the dividend already obtained to form the new dividend, 0000994009, and divide it by the trial divisor 600,000. 600,000 is contained in 0000994009 once, so we place 1 as the next figure in the root, and also add it to the trial divisor 600,000, thereby making the complete divisor 600,001. Multiply the complete divisor 600,001 by 1, the sixth figure in the root, and subtract the result obtained from the dividend. The remainder is 394,008, to which we annex the next period, 00, to form the next new dividend, or 39,400,800. Add the sixth figure of the root, or 1, to the divisor 600,001, and annex a cipher, thus obtaining 6,000,020 as the next trial divisor. Dividing 39,400,800 by 6,000,020, we find 6 to be the next figure of the root. Adding this last

figure, 6, to the trial divisor, we obtain 6,000,026 for our next complete divisor, which, multiplied by the last figure of the root, or 6, gives 36,000,156, which write under 39,400,800 and subtract. Since there is a remainder, it is clearly evident that the given power is not a perfect square, so we place + after the root.

In this problem, there are *seven* periods—four in the whole number and three in the decimal; hence, there will be *seven* figures in the root, *four* figures constituting the whole number and three figures the decimal of the root. Hence, $\sqrt{9,000,099.4009} = 3,000.016+$. Ans.

(*c*)

```
 3            √.00'12'25 = .035.  Ans.
 3             00
 ——            ————
 60              12
  5               9
 ——             ————
 65               325
                  325
                  ————
```

Pointing of periods, we find that the first period is composed of ciphers; hence, the first figure of the root will be a cipher. No further explanation is necessary, since this problem is solved in a manner exactly similar to the problem given in Art. **255.** Since there are *three* decimal periods in the power, there will be three decimal figures in the root.

(*d*)

```
 1            √1'07'95.21 = 103.9   Ans.
 1             1
 ——            ——————
 20              0795
  0               609
 ———             —————
 200              18621
   3              18621
 ———              —————
 203
   3
 ————
 2060
    9
 ————
 2069
```

(*e*)

2	$\sqrt{7'30'08.05} = 270.2+$ Ans.
2	4
40	330
7	329
47	10805
7	10804
5400	1
2	
5402	

(*f*) $\sqrt{9} = 3.$ Ans.

(*g*)

9	$\sqrt{.90'00'00} = .948+$ Ans.
9	81
180	900
4	736
184	16400
4	15104
1880	1296
8	
1888	

In Art. **258,** we see that every period of a decimal must consist of two figures when extracting the square root. We must, therefore, annex one cipher to .9 to form the first period.

(118) (*a*)

(1)	(2)	
6	36	$\sqrt[3]{.327'680'000} = .689+$ Ans.
6	72	216
12	10800	111680
6	1504	98432
180	12304	13248000
8	1568	12650769
188	1387200	597231
8	18441	
196	1405641	
8		
2040		
9		
2049		

EXPLANATION.—(1) When extracting the *cube* root, we divide the power into periods of three figures each. Always begin at the decimal point and proceed to the *left* in pointing off the whole number, and to the *right* in pointing off the decimal. In this power, $\sqrt[3]{.32768}$, a cipher must be annexed to 68 to complete the decimal period. Cipher periods may now be annexed until the root has as many figures as desired.

(2) We find by trial that the largest number whose cube is contained in the first period 327 is 6. Write 6 as the first figure of the root, also at the extreme left at the head of column (1). Multiply the 6 in column (1) by the first figure of the root 6, and write the product 36 at the head of column (2). Multiply the number in column (2) by the first figure of the root 6, and write the product 216 under the figures in the first period. Subtract and bring down the next period 680, annexing it to the remainder 111, thereby obtaining 111,680 for a new dividend. Add the first figure of the root 6 to the number in column (1), obtaining 12, which we call the *first correction;* multiply the first correction 12 by the first figure of the root and we obtain 72 as the product, which, added to 36 of column (2), gives 108. Add the first figure of the root to the first correction, and we obtain 18 as the *second correction.* To this annex *one* cipher, and annexing two ciphers to 108, we have 10,800 for the trial divisor. Dividing the dividend by the trial divisor, we see that it is contained about 8 times, so we write 8 as the second figure of the root. Adding the second figure of the root to 180, we obtain 188. This, multiplied by the second figure of the root 8, equals 1,504, which, added to the trial divisor 10,800, forms the *complete divisor*, 12,304. Multiplying the complete divisor 12,304 by 8, the second figure of the root, gives 98,432. Write 98,432 under the dividend 111,680; subtract, and there is a remainder of 13,248. To this remainder annex the next period, 000, thereby obtaining 13,248,000 for the next new dividend.

(3) Adding the second figure of the root 8 to the number in column (1), 188, we have 196 for the *first new correction.* This, multiplied by the second figure of the root 8, gives 1,568. Adding this product to the last complete divisor gives 13,872. Adding the second figure of the root, 8, to the first new correction, 196, we obtain 204 for the *second new correction.* Annexing one cipher to 204 and two ciphers to 13,872, we obtain 1,387,200 for the new trial divisor. Dividing the dividend by the trial divisor 1,387,200, we see that it is contained about 9 times. Write 9 as the third figure of the root. Add the third figure of the root, 9, to the last number in column (1), 2,040, thereby obtaining 2,049. This, multiplied by 9, the third figure of the root, equals 18,441, which, added to the trial divisor 1,387,200, forms the complete divisor 1,405,641. Multiplying the complete divisor by the third figure of the root 9, and subtracting, we have a remainder of 597,231. The answer, then, is .689 +. There are as many decimal places in the root as there are decimal periods in the power, or three decimal places, in this case.

(*b*)

(1)	(2)
4	16
4	32
8	4800
4	244
120	5044
2	
122	

$\sqrt[3]{74'088} = 42$ Ans.

```
 64
 10088
 10088
```

(*c*)

$$\sqrt[3]{92'416.000'000'000'000} = 45.2115+ \text{ Ans.}$$

(1)	(2)	
4	16	64
4	32	28416
8	4800	27125
4	625	1291000
120	5425	1220408
5	650	70592000
125	607500	61304761
5	2704	9287239000
130	610204	6131967931
5	2708	3155271069000
1350	61291200	3066085689875
2	13561	89185379125
1352	61304761	
2	13562	
1354	6131832300	
2	135631	
13560	6131967931	
1	135632	
13561	613210356300	
1	6781675	
13562	613217137975	
1		
135630		
1		
135631		
1		
135632		
1		
1356330		
5		
1356335		

Since *four cipher periods* were annexed, we must point off *four* decimal figures in the root.

(*d*)

$\sqrt[3]{.373'248} = .72$ Ans

(1)	(2)	
7	49	343
7	98	30248
14	14700	30248
7	424	
210	15124	
2		
212		

(*e*)

$\sqrt[3]{1'758.416'743} = 12.07$ Ans.

(1)	(2)	
1	1	1
1	2	758
2	300	728
1	64	30416743
30	364	30416743
2	68	
32	4320000	
2	25249	
34	4345249	
2		
3600		
7		
3607		

(*f*)

$\sqrt[3]{1'191'016} = 106$ Ans

(1)	(2)	
1	1	1
1	2	191016
2	30000	191016
1	1836	
300	31836	
6		
306		

(*g*) In Arts. **279** and **280**, we find that when the given number is in the form of a fraction, and it is required to find some root of it, the simplest method is to reduce the fraction to its equivalent decimal and then extract the required root of the decimal.

Thus, $\sqrt[3]{\frac{4}{32}} = ?$ $\frac{4}{32} = .125$, since

$$
\begin{array}{r}
32\,)\,4.000\,(\,.125 \\
\underline{32} \\
80 \\
\underline{64} \\
160 \\
\underline{160}
\end{array}
$$

(1)	(2)	
5	25	$\sqrt[3]{.125} = .5.$
		$\underline{125}$

Hence, $\sqrt[3]{\frac{4}{32}} = 5.$ Ans.

(*h*) (See Art. **279.**)

$\sqrt[3]{\frac{27}{256}} = ?$ $\frac{27}{256} = .10546875$, since

$$
\begin{array}{r}
256\,)\,27.00000000\,(\,.10546875 \\
\underline{256} \\
1400 \\
\underline{1280} \\
1200 \\
\underline{1024} \\
1760 \\
\underline{1536} \\
2240 \\
\underline{2048} \\
1920 \\
\underline{1792} \\
1280 \\
\underline{1280}
\end{array}
$$

(1)
4
4
8
4
120
7
127
7
134
7
1410
2
1412

(2)
16
32
4800
889
5689
938
662700
2824
665524

$\sqrt[3]{.105'468'750} = .472+$ Ans.
64
41468
39823
1645750
1331048
314702

(119)

(1)
1
1
2
1
30
2
32
2
34
2
360
5
365
5
370
5
3750
9
3759
9
3768
9
37770
9
37779

(2)
1
2
300
64
364
68
43200
1825
45025
1850
4687500
33831
4721331
33912
475524300
340011
475864311

$\sqrt[3]{2.000'000'000'000} = 1.2599+$ Ans.
1
1000
728
272000
225125
46875000
42491979
4383021000
4282771599
100249401

In this problem it was necessary to annex *four cipher periods*, in order to carry the decimal of the root to the required number of decimal places, or four.

(120)

(1)	(2)	
1	1	$\sqrt[3]{3.000'000'000} = 1.442+$ Ans
1	2	1
2	300	2000
1	136	1744
30	436	256000
4	152	241984
34	58800	14016000
4	1696	12458888
38	60496	1557112
4	1712	
420	6220800	
4	8644	
424	6229444	
4		
428		
4		
4320		
2		
4322		

In this problem it was necessary to annex *three* cipher periods, in order to carry the decimal of the root to the required number of decimal places, or three.

(121)

(*a*)		(*b*)	
1	$\sqrt{1'23.21} = 11.1$ Ans.	1	$\sqrt{1'14.92'10} = 10.72+$ Ans
1	1	1	1
20	23	200	1492
1	21	7	1449
21	221	207	4310
1	221	7	4284
220		2140	26
1		2	
221		2142	

(c)

7	$\sqrt{50'26'81} = 709$ Ans.
7	49
140	12681
0	12681
1400	
9	
1409	

(d)

2	$\sqrt{.00'04'12'09} = .0203$ Ans.
2	00
400	04
3	4
403	1209
	1209

(122) (a)

$\sqrt[3]{.006'500'000} = .186+$ Ans.

(1)	(2)	
1	1	1
1	2	5500
2	300	4832
1	304	668000
30	604	602856
8	368	65144
38	97200	
8	3276	
46	100476	
8		
540		
6		
546		

(b)

$\sqrt[3]{.021'000} = .27+$ Ans

(1)	(2)	
2	4	8
2	1	13000
4	1200	11683
2	469	1317
60	1669	
7		
67		

(c) $\sqrt[3]{8'036'054'027} = 2003$ Ans.

(1)	(2)	
2	4	8
2	8	036054027
4	12000000	36054027
2	18009	
6000	12018009	
3		
6003		

(d) $\sqrt[3]{.000'004'096} = .016$ Ans.

(1)	(2)	
1	1	000
1	2	004
2	300	1
1	216	3096
30	516	3096
6		
36		

(e) $\sqrt[3]{17.000'000} = 2.57+$ Ans.

(1)	(2)	
2	4	8
2	8	9000
4	1200	7625
2	325	1375000
60	1525	1349593
5	350	25407
65	187500	
5	5299	
70	192799	
5		
750		
7		
757		

(123) (*a*) $\sqrt{\frac{1,225}{5,476}} = \frac{\sqrt{1,225}}{\sqrt{5,476}}$.

$$
\begin{array}{r}
3 \\
3 \\
\hline
60 \\
5 \\
\hline
65
\end{array}
\qquad
\begin{array}{l}
\sqrt{12'25} = 35 \\
\;\;9 \\
\hline
\;\;325 \\
\;\;325 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
7 \\
7 \\
\hline
140 \\
4 \\
\hline
144
\end{array}
\qquad
\begin{array}{l}
\sqrt{54'76} = 74 \\
\;\;49 \\
\hline
\;\;\;576 \\
\;\;\;576 \\
\hline
\end{array}
$$

Hence, $\sqrt{\frac{1,225}{5,476}} = \frac{35}{74}$. Ans.

(*b*)

$$
\begin{array}{r}
5 \\
5 \\
\hline
100 \\
8 \\
\hline
108
\end{array}
\qquad
\begin{array}{l}
\sqrt{.33'64} = .58 \\
\;\;25 \qquad \text{Ans.} \\
\hline
\;\;\;864 \\
\;\;\;864 \\
\hline
\end{array}
$$

(*c*)

$$
\begin{array}{r}
3 \\
3 \\
\hline
60 \\
1 \\
\hline
61 \\
1 \\
\hline
620 \\
6 \\
\hline
626 \\
6 \\
\hline
6320 \\
2 \\
\hline
6322 \\
2 \\
\hline
63240 \\
2 \\
\hline
63242
\end{array}
\qquad
\begin{array}{l}
\sqrt{.10'00'00'00'00} = .31622+ \\
\;\;9 \qquad\qquad\qquad\qquad \text{Ans.} \\
\hline
\;\;100 \\
\;\;\;61 \\
\hline
\;\;\;3900 \\
\;\;\;3756 \\
\hline
\;\;\;\;14400 \\
\;\;\;\;12644 \\
\hline
\;\;\;\;\;175600 \\
\;\;\;\;\;126484 \\
\hline
\;\;\;\;\;\;49116
\end{array}
$$

(*d*) $25.0\frac{3}{4} = 25.075.$

5	$\sqrt{25.07'50'00'00'00} = 5.00749+$ Ans.
5	25
10000	075000
7	70049
10007	495100
7	400576
100140	9452400
4	9013401
100144	438999
4	
1001480	
9	
1001489	

(*e*) $.000\frac{4}{9} = .0004444444+.$

2	$\sqrt{.00'04'44'44'44} = .02108+$ Ans
2	00
40	04
1	4
41	44
1	41
4200	34444
8	33664
4208	780

(124) $11.7 : 13 :: 20 : x$
$11.7x = 13 \times 20$
$11.7x = 260$

The product of the means equals the product of the extremes.

$x = \frac{260}{11.7}$

```
11.7)260.000(22.22+  Ans
     234
      260
      234
       260
       234
        260
        234
         26
```

(125) (*a*) $20 + 7 : 10 + 8 :: 3 : x.$

$27 : 18 :: 3 : x$

$27x = 18 \times 3$

$27x = 54$

$x = \frac{54}{27} = 2.$ Ans.

(*b*) $(12)^2 : (100)^2 :: 4 : x$

$144 : 10{,}000 :: 4 : x$

$144x = 10{,}000 \times 4$

$144x = 40{,}000$

$$\begin{array}{r} 12 \\ 12 \\ \hline 144 \end{array} \qquad \begin{array}{r} 100 \\ 100 \\ \hline 10000 \end{array}$$

$x = \frac{40{,}000}{144}$ 144)40000.0(277.7+ Ans.

$$\begin{array}{l} 288 \\ \hline 1120 \\ 1008 \\ \hline \ \ 1120 \\ \ \ 1008 \\ \hline \ \ \ \ 1120 \\ \ \ \ \ 1008 \\ \hline \end{array}$$

(126) (*a*) $\frac{4}{x} = \frac{7}{21}$ is equivalent to $4 : x :: 7 : 21$. The product of the means equals the product of the extremes. Hence,

$7x = 4 \times 21$

$7x = 84$

$x = \frac{84}{7}$, or 12. Ans.

In like manner,

(*b*) $\frac{x}{24} = \frac{8}{16}$ is equivalent to $x : 24 :: 8 : 16$.

$16x = 24 \times 8$

$16x = 192$

$x = \frac{192}{16} = 12.$ Ans.

(*c*) $\frac{2}{10} = \frac{x}{100}$ is equivalent to $2 : 10 :: x : 100$

$$10\,x = 2 \times 100$$
$$10\,x = 200$$
$$x = \frac{200}{10} = 20. \quad \text{Ans.}$$

(*d*) $\frac{15}{45} = \frac{60}{x}$ is equivalent to $15 : 45 :: 60 : x$

$$15\,x = 45 \times 60$$
$$15\,x = 2{,}700$$
$$x = \frac{2{,}700}{15} = 180. \quad \text{Ans.}$$

(*e*) $\frac{10}{150} = \frac{x}{600}$ is equivalent to $10 : 150 :: x : 600.$

$$150\,x = 10 \times 600$$
$$150\,x = 6{,}000$$
$$x = \frac{6{,}000}{150} = 40. \quad \text{Ans}$$

(127) $x : 5 :: 27 : 12.5$

```
               5
         ----------
    12.5)135.0(10 4/5.  Ans.
          125
         -----
           100    4
           --- = -.
           125    5
```

Hence, $x = 10\frac{4}{5}$. Ans.

(128) $45 : 60 :: x : 24$

$$60\,x = 45 \times 24$$
$$60\,x = 1{,}080$$
$$x = \frac{1{,}080}{60} = 18. \quad \text{Ans}$$

(129) $x : 35 :: 4 : 7$

$$7\,x = 35 \times 4$$
$$7\,x = 140$$
$$x = \frac{140}{7} = 20. \quad \text{Ans.}$$

(130) $9 : x :: 6 : 24$

$$6\,x = 9 \times 24$$
$$6\,x = 216$$
$$x = \frac{216}{6} = 36. \quad \text{Ans.}$$

(131) $\sqrt[3]{1,000} : \sqrt[3]{1,331} :: 27 : x$ $\qquad \sqrt[3]{1,000} = 10$

$\sqrt[3]{1,331} = 11$

$10 : 11 :: 27 : x$

$10x = 297$

$x = \frac{297}{10} = 29.7.$ Ans.

(1)	(2)	
1	1	1'331(11
1	2	1
2	300	331
1	31	331
30	331	
1		
31		

(132) $64 : 81 :: 21^2 : x^2$. Extracting the square root of each term of any proportion does not change its value, so we find that $\sqrt{64} : \sqrt{81} :: \sqrt{21^2} : \sqrt{x^2}$ is the same as

$$8 : 9 :: 21 : x$$
$$8x = 189$$
$$x = 23.625. \quad \text{Ans.}$$

(133) $7 + 8 : 7 :: 30 : x$ is equivalent to $15 : 7 :: 30 : x$.

$$15x = 7 \times 30$$
$$15x = 210$$
$$x = \frac{210}{15} = 14. \quad \text{Ans.}$$

(134) $3\frac{1}{2}$ ft. $= 3.5$ ft., since $\frac{1}{2} = .5$.

$6\frac{3}{4}$ ft. $= 6.75$ ft., since $\frac{3}{4} = .75$.

Consider the question "What do we wish to find?" In this case it is "pounds." We know that a piece of shafting 3.5 ft. long weighs 37.45 lb., and we wish to know the weight of a piece of shafting $6\frac{3}{4}$ ft. long. It is evident that the weight of a piece of shafting 6.75 ft. long bears the same relation to the weight of a piece 3.5 ft. long that 6.75 ft. bears to 3.5 ft. Letting x occupy any place in the propor-

tion, we have the following, the value of x being the same in each. Thus,

(*a*) 3.5 ft. : 6.75 ft. :: 37.45 lb. : x lb.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(*b*) 6.75 ft. : 3.5 ft. :: x lb. : 37.45 lb.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(*c*) x lb. : 37.45 lb. :: 6.75 ft. : 3.5 ft.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(*d*) 37.45 lb. : x lb. :: 3.5 ft. : 6.75 ft.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(135) In this problem we are required to find "degrees." In examples of this kind, it is better to write the statement down as a simple direct proportion, and then make what changes are necessary. Written as a direct proportion, we would have 6 : 12 :: 24 : x. But, since the heat varies *inversely*, the proportion is an inverse one. Hence, inverting one of the couplets, we have 12 : 6 :: 24 : x. The example also states that the effect of the heat upon the thermometer varies inversely as the *square of the distance* from the burning body. Consequently, the two distances, 6 ft. and 12 ft., in the above proportion must be squared, and we have $12^2 : 6^2 :: 24 : x$. $12^2 = 144$ and $6^2 = 36$. Therefore, 144 : 36 :: 24 : x,

$$\text{or } x = \frac{36 \times 24}{144} = 6^\circ. \quad \text{Ans.}$$

(136) This problem may be solved by very simple reasoning, thus: If sound travels at the rate of 6,160 feet in $5\frac{1}{2}$ seconds, in 1 minute, which is $10\frac{10}{11}$ times $5\frac{1}{2}$ seconds

$\left(60 \div \frac{11}{2} = 60 \times \frac{2}{11} = \frac{120}{11} = 10\frac{10}{11}\right)$, it will travel $10\frac{10}{11} \times$ 6,160 ft., or 67,200 ft. Ans.

$$\begin{array}{r} 6160 \\ 10\frac{10}{11} \\ \hline 61600 \\ 5600 \\ \hline 67200 \end{array} \text{ ft. Ans.} \qquad \frac{10}{11} \times 6{,}160 = \frac{61{,}600}{11} = 5{,}600.$$

Or, if sound travels at the rate of 6,160 feet in $5\frac{1}{2}$ seconds, in 1 second it will travel as many feet as $5\frac{1}{2}$ (5.5) is contained times in 6,160 feet, or 1,120 feet, and in 1 minute, or 60 seconds, it will travel $1{,}120 \times 60$, or 67,200 feet.

Or, it may be solved by proportion, as follows: In this problem it states that sound travels at the rate of 6,160 feet in $5\frac{1}{2}$ (5.5) seconds. We wish to know how far it will travel in 1 minute, which is equivalent to 60 seconds. It is evident that the distance it travels in 1 minute, or 60 seconds, bears the same relation to the distance it travels in 5.5 seconds that 1 minute, or 60 seconds, bears to 5.5 seconds. Letting x occupy any place in the proportion, we have the following, the value of x being the same in each. Thus,

(*a*) 6,160 feet : x feet :: 5.5 seconds : 60 seconds,

$$\text{or } x = \frac{6{,}160 \times 60}{5.5} = \frac{369{,}600}{5.5}$$

$$\begin{array}{r} 5.5)369600.0(67200 \text{ ft. Ans.} \\ 330\phantom{000.0(67200 \text{ ft. Ans.}} \\ \hline 396\phantom{00.0(67200 \text{ ft. Ans.}} \\ 385\phantom{00.0(67200 \text{ ft. Ans.}} \\ \hline 110\phantom{0.0(67200 \text{ ft. Ans.}} \\ 110\phantom{0.0(67200 \text{ ft. Ans.}} \\ \hline \end{array}$$

(*b*) x feet : 6,160 feet :: 60 seconds : 5.5 seconds,

$$\text{or } x = \frac{6{,}160 \times 60}{5.5} = 67{,}200 \text{ feet. Ans.}$$

(*c*) 60 seconds : 5.5 seconds :: x feet : 6,160 feet,

$$\text{or } x = \frac{6{,}160 \times 60}{5.5} = 67{,}200 \text{ feet. Ans.}$$

(*d*) 5.5 seconds : 60 seconds :: 6,160 feet : x feet,

$$\text{or } x = \frac{6{,}160 \times 60}{5.5} = 67{,}200 \text{ feet. Ans.}$$

(137) We will first reduce 8 hr. 40 min. to minutes. 8 hr. + 40 min. = (8 × 60 min.) + 40 min. = 520 min. In this problem we are required to find "time." We know that a railway train runs 444 miles in 520 minutes, and we want to know how long it will take it to run 1,060 miles at the same rate of speed. It is evident that the time it requires to run 1,060 miles bears the same relation to the time it takes to run 444 miles that 1,060 miles bears to 444 miles. Letting x occupy any place in the proportion, we have the following, the value of x being the same in each. Thus,

(*a*) 1,060 miles : 444 miles :: x min. : 520 min.,

$$\text{or } x = \frac{1{,}060 \times 520}{444} = \frac{551{,}200}{444} =$$

```
 1060
  520
-----
21200
5300
-----
551200
```

```
444)551200.00(1241.44+ min.
    444
    ---
    1072
     888
    ----
     1840
     1776
     ----
       640
       444
       ---
       1960
       1776
       ----
        1840
        1776
        ----
          64
```

Reducing 1,241.44 min. to hours by dividing by 60, we have

$$60)1241.44(20 \text{ hr. } 41.44 \text{ min. Ans.}$$
$$\underline{120}$$
$$41$$

(*b*) 444 miles : 1,060 miles :: 520 min. : x min.

$$x = \frac{1,060 \times 520}{444} = 1,241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min. Ans.}$$

(*c*) x min. : 520 min. :: 1,060 miles : 444 miles.

$$x = \frac{1,060 \times 520}{444} = 1,241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min. Ans.}$$

(*d*) 520 min. : x min. :: 444 miles : 1,060 miles.

$$x = \frac{1,060 \times 520}{444} = 1,241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min. Ans.}$$

(138) This is an *inverse* ratio since it will take a pump discharging 85 gal. per minute a *longer* time to fill the tank than one discharging 135 gal. per minute. The direct proportion would be

135 gal. : 85 gal. :: 38 min. : x min.

In order to make the proportion inverse, we must invert one of the couplets, and we have

135 gal. : 85 gal. : x min. : 38 min.

$$x = \frac{135 \times 38}{85} = 60\frac{6}{17} \text{ min. Ans.}$$

$$\begin{array}{r} 135 \\ \times\ 38 \\ \hline 1080 \\ 405 \\ \hline 85)5130(60\frac{6}{17} \\ 510 \\ \hline 30 \end{array}$$

$$\frac{30}{85} = \frac{6}{17}$$

(139) 8 lb. + 8 lb. + 80 lb. = 96 lb., or the number of pounds in the mixture. The relation between the amount of copper which 36 lb. of this mixture will contain, and the amount which 96 lb. contain is the same as 32 : 96; whence, the proportion,

$$32 \text{ lb.} : 96 \text{ lb.} :: x : 8 \text{ lb.},$$

```
96)256(2
   192
   ---
    64   2
    -- = -
    96   3
```

$$\text{or } x = \frac{32 \times 8}{96} = \frac{256}{96} = 2\frac{2}{3} \text{ lb. Ans.}$$

(140) This will be an *inverse* ratio, since the *larger* wheel will turn a less number of times than the smaller wheel in going a certain distance. The direct proportion is

$$12.56 \text{ ft.} : 15.7 \text{ ft.} :: 520 : x.$$

Inverting one of the couplets,

$$15.7 \text{ ft.} : 12.56 \text{ ft.} :: 520 : x,$$

$$\text{or } x = \frac{12.56 \times 520}{15.7} = 416 \text{ times. Ans.}$$

```
            520
          12.56
          -----
           3120
          2600
         1040
         520
        -------
15.7 ) 6531.20 ( 416 times
       628
       ---
        251
        157
        ---
         942
         942
         ---
```

(141) Before forming the proportion, we will combine the three simple ratios into one by reducing them all to the same denomination. A cistern 28 feet long, 12 feet wide, and 10 feet deep contains $28 \times 12 \times 10 = 3,360$ cubic feet.

Again, a cistern 20 ft. long, 17 ft. wide, and 6 ft. deep contains $20 \times 17 \times 6 = 2{,}040$ cubic feet. What do we wish to find? In this case it is "barrels." We know that a cistern containing 3,360 cubic feet holds 798 barrels of water, and we want to know how many barrels of water a cistern containing 2,040 cubic feet will hold. The number of barrels that a cistern containing 2,040 cubic feet will hold bears the same relation to the number of barrels that a cistern containing 3,360 cubic feet holds as 2,040 cubic feet bears to 3,360 cubic feet. Hence,

$$2{,}040 \text{ cu. ft.} : 3{,}360 \text{ cu. ft.} :: x \text{ bbl.} : 798 \text{ bbl.},$$

or $$x = \frac{\overset{85}{\cancel{2{,}040}} \times \overset{57}{\cancel{798}}}{\underset{\underset{10}{\cancel{140}}}{\cancel{3{,}360}}} = \frac{85 \times 57}{10} = 484.5 = 484\tfrac{1}{2} \text{ bbl.} \quad \text{Ans.}$$

MENSURATION AND USE OF LETTERS IN FORMULAS.

(QUESTIONS 142-218.)

(142) Substituting for D, x, B, and i their values,

$$C = \frac{D - x}{B + i} = \frac{120 - 12}{10 + 3.5} = \frac{108}{13.5} = 8. \quad \text{Ans.}$$

A line between two numbers signifies that the one above the line, or numerator, is to be divided by the one below the line, or denominator.

(143) Substituting for A, h, D, and x their values,

$$\frac{A\,h + D}{2x + 6} = \frac{(5 \times 200) + 120}{(2 \times 12) + 6} = \frac{1{,}000 + 120}{24 + 6} = \frac{1{,}120}{30} = 37\tfrac{1}{3}.$$

$$37\tfrac{1}{3} + D = 37\tfrac{1}{3} + 120 = 157\tfrac{1}{3}. \quad \text{Ans.}$$

When there is no sign between the letters, multiplication is understood.

(144) Substituting for B, h, A, x, and i their values,

$$r = \frac{3.246 \times B \times h}{\frac{A\,x + h}{A\,i - B}} = \frac{3.246 \times 10 \times 200}{\frac{(5 \times 12) + 200}{(5 \times 3.5) - 10}} = \frac{6{,}492}{\frac{260}{7.5}} =$$

$$6{,}492 \div \frac{260}{7.5} = 6{,}492 \times \frac{7.5}{260} = 187.269 +. \quad \text{Ans.}$$

(145) Substituting for A, D, i, and B their values,

$$v = \sqrt{\frac{A\,D}{i\,B + 1.5}} = \sqrt{\frac{5 \times 120}{(3.5 \times 10) + 1.5}} = \sqrt{\frac{600}{36.5}} =$$

$$\sqrt{16.4383} = 4.05 +. \quad \text{Ans.}$$

The square root sign extends over both numerator and denominator, thus indicating that the square root of the entire fraction is to be extracted.

(146) Substituting for B, x, h, and A their values,

$$u = \sqrt[3]{\frac{Bx}{.00018\,h\,(A^2 - x)}} = \sqrt[3]{\frac{10 \times 12}{.00018 \times 200 \times (5^2 - 12)}} =$$

$$\sqrt[3]{\frac{120}{.036 \times (25 - 12)}} = \sqrt[3]{\frac{120}{.036 \times 13}} = \sqrt[3]{\frac{120}{.468}} =$$

$$\sqrt[3]{256.41} + = 6.35 +. \quad \text{Ans.}$$

(147) Substituting for h, D, and A their values,

$$f = \frac{10(h-D)^2}{\sqrt[3]{D+A}} = \frac{10\,(200-120)^2}{\sqrt[3]{120+5}} = \frac{10 \times 80^2}{\sqrt[3]{125}} = \frac{64,000}{5} = 12,800. \quad \text{Ans.}$$

(148) Substituting for B, A, and D their values,

$$g = \frac{(B-A)^2 - \sqrt[3]{D+A}}{A^3 - (1+D)} = \frac{(10-5)^2 - \sqrt[3]{120+5}}{5^3 - (1+120)} =$$

$$\frac{5^2 - \sqrt[3]{125}}{125 - 121} = \frac{25-5}{4} = \frac{20}{4} = 5. \quad \text{Ans.}$$

(149) Substituting for A, B, and h their values,

$$k = \sqrt{\frac{AB^2}{\sqrt[3]{Ah}}} = \sqrt{\frac{5 \times 10^2}{\sqrt[3]{5 \times 200}}} = \sqrt{\frac{5 \times 100}{\sqrt[3]{1,000}}} = \sqrt{\frac{500}{10}} =$$

$$\sqrt{50} = 7.071 +. \quad \text{Ans.}$$

(150) Substituting for A, h, D, x, and B their values,

$$T = \sqrt{\frac{A^2\left[490 + \frac{(hx)^2}{D^2}\right]}{h + \frac{x}{D}(A^2 - B)^2}} = \sqrt{\frac{5^2\left[490 + \frac{(200 \times 12)^2}{120^2}\right]}{200 + \frac{12}{120}(5^2 - 10)^2}} =$$

$$\sqrt{\frac{25\,(490 + 400)}{200 + (\frac{1}{10} \times 225)}} = \sqrt{\frac{25 \times 890}{200 + 22.5}} = \sqrt{\frac{22,250}{222.5}} =$$

$$\sqrt{100} = 10. \quad \text{Ans.}$$

(151) When one straight line meets another straight line, two angles are formed which together equal 180°. Hence, if one of the angles $= 152°\ 3'$, the other angle $= 180° - 152°\ 3'$, or

$$\begin{array}{rr} & 180° = 179°\ 60' \\ \text{subtracting,} & 152°\ \ 3' \\ \hline & 27°\ 57'. \ \text{Ans.} \end{array}$$

(152) There are 60 seconds in one minute and 60 minutes in one degree; therefore, $140° = 140 \times 60 = 8{,}400$ minutes; $8{,}400' + 17' = 8{,}417'$; $8{,}417' = 8{,}417 \times 60 = 505{,}020$ seconds, and $505{,}020'' + 10'' = 505{,}030''$. Ans.

(153) See Arts. **359** and **360.**

(154) (*a*) $240 \div 60 = 4$, the number of degrees. Ans.
(*b*) $240 \times 60 = 14{,}400$, the number of seconds. Ans.

(155) See Arts. **355-357.**

(156) See Art. **369.** If the rectangle has the same base and altitude, it would have the same area.

(157) No, since the sum of the three shorter sides is not greater than the fourth side.

(158) Since the area is to be found in square inches, the $2\frac{1}{2}$ feet must be reduced to inches. $2\frac{1}{2}$ ft. = 30 in. Area $= 30 \times 11\frac{1}{2} = 345$ sq. in. Ans.

(159) Since there are 144 sq. in. in 1 sq. ft., the area of the zinc in the last example $= \frac{345}{144} = 2.396$ sq. ft. The weight per square foot $= \frac{5.25}{2.396} = 2.19 +$ lb. Ans.

(160) It will take $1\frac{1}{2}$ boards to reach lengthways of the room. Since the room is 15 feet wide, and each board is 5 *inches* wide, it will take $15 \div \frac{5}{12} = 36$ boards, laid side by side, to extend across the width of the room. Hence, number of boards required $= 36 \times 1\frac{1}{2} = 54$. Ans.

(161) By the rule for the area of a trapezoid, area of the land $= \frac{9 + 6}{2} \times 16 = 120$ square rods. Since there are 160 square rods in one acre, 120 square rods $= \frac{120}{160} = \frac{3}{4}$ of an acre. Ans.

(162) The total area of the floor of the station $= 55 \times 58$ ft. $= 3{,}190$ sq. ft. $- 25 \times 26$ ft. $= 650$ sq. ft., the area represented by the lower right-hand corner of the

figure. Hence, total area of floor = 3,190 − 650 = 2,540 sq. ft.

From this we have to deduct the following areas:

2 boilers	$= 2 \times 8 \times 19$	$= 304.$	sq. ft.
Feed pump	$= 2\frac{1}{2} \times 5$	$= 12.5$	sq. ft.
2 engines	$= 2 \times 4\frac{1}{2} \times 10$	$= 90$	sq. ft.
2 dynamos	$= 2 \times 5\frac{1}{2} \times 6\frac{1}{2}$	$= 71.5$	sq. ft.
Switchboard	$= \dfrac{10 \times 3.5}{12}$	$= 2.92$	sq. ft.
		480.92	sq. ft.

The unoccupied floor space, therefore, =

$2,540 - 480.92 = 2,059.08$ sq. ft. Ans.

(163) The length of the walk, neglecting the corners, that is, the distance $a\,b + b\,c + c\,d + d\,a$, Fig. 1, $= 2 \times 528 + 2 \times 352 = 1,760$ ft. The width = 10 ft.; area $= 1,760 \times 10 = 17,600$ sq. ft. The area of the four corners $= 4 \times 10^2 = 400$ sq. ft. Total area of walk $= 17,600 + 400 = 18,000$ sq. ft. Hence the area in sq. yds. $= \dfrac{18,000}{9} = 2,000.$ Ans.

FIG. 1.

(164) The area of the sides of the room $= 2 \times 20 \times 90 = 3,600$ sq. ft.; area of ends $= 2 \times 20 \times 50 = 2,000$ sq. ft. Total area of walls $= 3,600 + 2,000 = 5,600$ sq. ft.

From this is to be deducted the following areas:

4 doors $= 4 \times 5\frac{1}{2} \times 10 = 220$ sq. ft.
14 windows $= 14 \times 5 \times 11 = 770$ sq. ft.

Length of baseboard, deducting the width of the 4 doors $= 2 \times 90 + 2 \times 50 - 4 \times 5\frac{1}{2} = 180 + 100 - 22 = 258$ feet; width of baseboard = 9 inches, or $\frac{3}{4}$ of a foot; area of baseboard $= 258 \times \dfrac{3}{4} = 193.5$ sq. ft. Hence, we have to deduct $220 + 770 + 193.5 = 1,183.5$ sq. ft.

Number of sq. ft. of plastering = 5,600 − 1,183.5 = 4,416.5 sq. ft.; number of sq. yd. of plastering = 4,416.5 ÷ 9 = 490.72. Ans.

(165) A triangle with three equal angles has three equal sides, and is, therefore, an equilateral triangle.

(166) A triangle with two equal angles has two equal sides, and is, therefore, an isosceles triangle.

(167) No, since the sum of the two shorter sides is not greater than the third side.

(168) (*a*) Draw a line $B\,D$ from the vertex perpendicular to the base, Fig. 2. It will divide the base into two equal parts, as shown. In the right-angled triangle $A\,B\,D$, the hypotenuse $A\,B = 6$, and side $A\,D = 3$; hence, by rule **50**, Art. **385**, $B\,D$, the altitude $= \sqrt{6^2 - 3^2} = \sqrt{27} = 5.196$ ft. Ans.

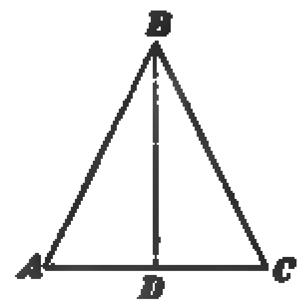
FIG. 2.

(*b*) Area $= \dfrac{6 \times 5.196}{2} = 15.588$ sq. ft. Ans.

(169) The sum of the three angles in any triangle = 2 right angles, or 180°. In the given triangle, the sum of two angles $= 23° + 32°\ 32' = 55°\ 32'$, and the third angle $= 180° - 55°\ 32'$, or

$$\begin{array}{rr} 180° = & 179°\ 60' \\ \text{subtracting,} & 55°\ 32' \\ \hline & 124°\ 28'. \end{array}\quad \text{Ans.}$$

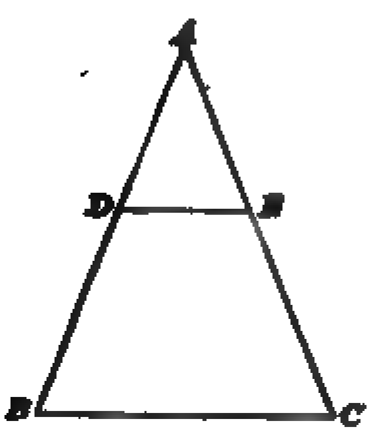
FIG. 3.

(170) In Fig. 3 we have the proportion $A\,D : D\,E :: A\,B : B\,C$, in which $A\,D = 10$ in., $A\,B = 24$ in., and $B\,C = 13\tfrac{1}{2}$ in., to find $D\,E$.

Substituting the given values,

$$10 : D\,E :: 24 : 13\tfrac{1}{2}, \text{ or}$$

$$D\,E = \frac{10 \times 13.5}{24} = 5.625 \text{ in.} \quad \text{Ans.}$$

(171) A line drawn diagonally from one corner to the opposite one would form the hypotenuse of a right-angled

triangle, whose two sides are 39 and 52 feet. By rule **49**, Art. **385**, the length of the diagonal $= \sqrt{52^2 + 39^2} = 65$ ft. Ans.

(172) Using rule **50**, Art. **385**, the required distance $= \sqrt{24^2 - 8^2} = \sqrt{576 - 64} = 22.627$ ft. $= 22$ ft. $7\frac{1}{2}$ in. +. Ans.

(173) See Art. **386**.

(174) (*a*) Using rule **52**, Art. **386**, length of base =

$$\frac{2 \times 200}{20} = 20 \text{ in. Ans.}$$

(*b*) A perpendicular let fall from the vertex to the base will divide the triangle into two equal right-angled triangles, of which the perpendicular side is 20 inches and the horizontal side 10 inches. Hence, according to rule **49**, Art. **385**, the hypotenuse $= \sqrt{20^2 + 10^2} = 22.36$ in. = the length of one of the equal sides of the given triangle. Ans.

(175) An equilateral heptagon has seven equal sides; hence, the sum of all the sides, or the perimeter, $= 7 \times 3 = 21$ in. Ans.

(176) A regular decagon has 10 equal sides; hence, the length of one of the sides $= 40 \div 10 = 4$ inches. Ans.

(177) A dodecagon has 12 sides, and by rule **53**, Art. **389**, we have interior angle $\frac{180 \times (12 - 2)}{12} = 150°$. Ans.

(178) Divide the pentagon into five equal isosceles triangles, as shown in Fig. 4, by drawing a line from the center to each angle. The area of one of the triangles, as $A\ B\ C$, $= 43 \div 5 = 8.6$ sq. in. We have given, therefore, the area and base $B\ C$ of the triangle $A\ B\ C$, to find its altitude $A\ D$. Using rule **52**, Art. **386**, $A\ D = \frac{8.6 \times 2}{5} = 3.44$ in., the perpendicular distance from the center to one side. Ans.

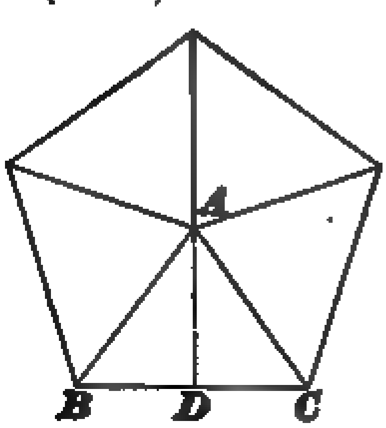

FIG. 4.

(179) See example, Art. **389.** The process is simply to find one of the angles of the polygon, and then to divide it by 2. By rule **53,** Art. **389,** one of the interior angles $= \frac{180 \times (8-2)}{8} = 135°$. This divided by $2 = 67\frac{1}{2}°$. Ans.

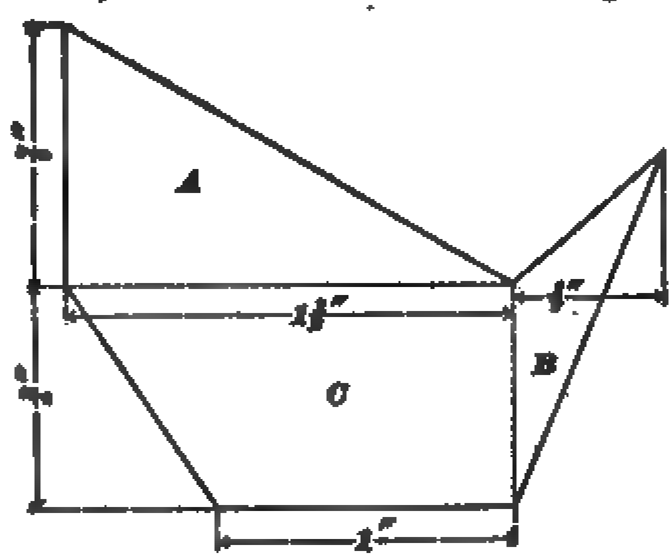

FIG. 5.

(180) Divide the figure into two triangles and one trapezoid, as shown in Fig. 5. The dimensions of the different parts can then be found by measurement. In obtaining the areas, it will be easier to use decimals than common fractions.

Area of triangle $A = \frac{1\frac{1}{2} \times \frac{7}{8}}{2} = \frac{1.5 \times .875}{2} = .656$ sq. in.

Area of triangle $B = \frac{\frac{3}{4} \times \frac{1}{2}}{2} = \frac{.75 \times .5}{2} = .188$ sq. in.

Area of trapezoid $C = \frac{1\frac{1}{2}+1}{2} \times \frac{3}{4} = \frac{1.5+1}{2} \times .75 = 1.25 \times .75 = .938$ sq. in.

Hence, the area of the whole figure $= .656 + .188 + .938 = 1.78 +$ sq. in. Ans.

(181) An angle inscribed in a circle is measured by one-half the intercepted arc. In this case, the angle intercepts one-fourth the circumference, and is measured by one-eighth the circumference, or by $360° \times \frac{1}{8} = 45°$. Hence, there are 45° in the angle. Ans.

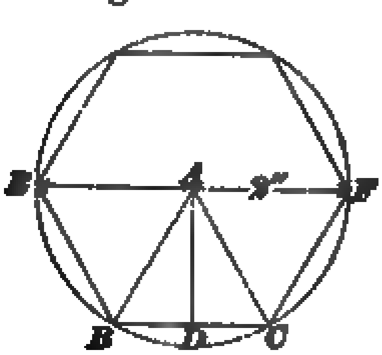

FIG. 6.

(182) Since this is a regular hexagon, it may be inscribed in a circle (Fig. 6), and the radius of the inscribing circle will be equal to one side of the hexagon. Since the diameter $EF = 2$ inches, the radii

$A\,B$ and $A\,C$, and the side $B\,C$ each = 1 inch, and the triangle $A\,B\,C$ is equilateral. Draw the line $A\,D$ perpendicular to the side $B\,C$; it will bisect $B\,C$. Then, in the right-angled triangle $A\,D\,B$, $A\,B = 1$, and $B\,D = \frac{1}{2}''$, to find $A\,D$. According to rule **50**, Art. **385**, $A\,D = \sqrt{1^2 - .5^2} = \sqrt{.75} = .866''$. Hence, the distance between two opposite sides of the hexagon $= A\,D \times 2 = .866 \times 2 = 1.732''$. Ans.

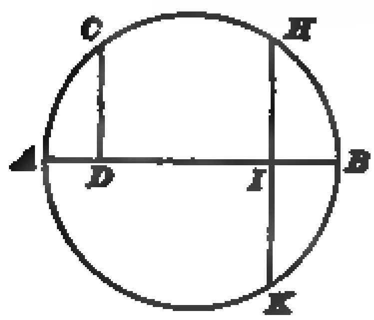

FIG. 7.

(183) In Fig. 7, we have the proportion $B\,I : H\,I :: H\,I : I\,A$, in which $B\,I = 6$, and $H\,I = \frac{1}{2}$ of $H\,K = \frac{18}{2} = 9$. Substituting, $6 : 9 :: 9 : I\,A$, or $I\,A = \frac{81}{6} = 13.5$ in. Hence, the diameter $A\,B = I\,A + B\,I = 13.5 + 6 = 19.5$ in. Ans.

(184) If the diameter $A\,B = 32\frac{1}{2}$ ft. and $I\,B = 8$ ft., $A\,I = 32\frac{1}{2} - 8 = 24\frac{1}{2}$ ft. Then, from the proportion of the last example, $8 : H\,I :: H\,I : 24.5$, whence $H\,I = \sqrt{8 \times 24.5} = \sqrt{196} = 14$ ft., and $H\,K = 2 \times 14 = 28$ ft. Ans. (See Art. **400**.)

(185) Diameter = circumference ÷ 3.1416. By rule **56**, Art. **403**, diameter of tree $= 7.854 \div 3.1416 = 2\frac{1}{2}$ ft. Ans.

(186) One mile = 5,280 feet. The circumference of the wheel in feet $= \frac{72 \times 3.1416}{12} = 18.8496$. (See rule **55**, Art. **402**.) Number of revolutions in going one mile $= 5{,}280 \div 18.8496 = 280.112$. Ans.

(187) Using rule **58**, Art. **405**, area = diameter squared $\times .7854$. $6.06^2 = 36.7236$; $36.7236 \times .7854 = 28.8427$ sq. in. Ans.

(188) By rule **59**, Art. **406**, inside diameter $= \sqrt{\frac{113.0976}{.7854}} = 12''$. Since the pipe is $\frac{3}{4}''$ thick, the outside diameter must be $\frac{3}{4}'' \times 2 = 1\frac{1}{2}''$ more, or $12 + 1\frac{1}{2} = 13\frac{1}{2}''$. Ans

(189) Since the radius of the circle = 6 in., its diameter = 12 in., and its circumference = $12 \times 3.1416 = 37.6992$ in. There are 360° in the circumference, and the length of an arc of $12° = 37.6992 \times \frac{12}{360} = 1.25664$ in. Ans.

(190) The area of a circle 22 inches in diameter $= 22^2 \times .7854 = 380.1336$ sq. in. (See rule **58**, Art. **405**.) Area of a circle 21 inches in diameter $= 21^2 \times .7854 = 346.3614$ sq. in. Hence, the area of a flat ring whose outside diameter = 22 in. and inside diameter = 21 in. is $380.1336 - 346.3614 = 33.7722$ sq. in. Ans.

(191) In the formula of rule **61**, Art. **408**, $\frac{4h^2}{3}\sqrt{\frac{D}{h} - .608}$, h = the height of the segment = 5 in., and D = the diameter of the circle = 56 in. Hence, the area of the segment $= \frac{4 \times 5^2}{3}\sqrt{\frac{56}{5} - .608} = \frac{100}{3}\sqrt{10.592} = 33\frac{1}{3} \times 3.255 =$ 108.5 sq. in. Ans.

(192) In Fig. 8, let $A\,C\,B\,D$ represent a section of the largest square bar that can be planed from the round bar. $A\,B$ and $C\,D$ each = 2 in., and in the right-angled triangle $A\,O\,C$, the sides $A\,O$ and $C\,O$ each = 1 in. The hypotenuse $A\,C = \sqrt{1^2 + 1^2} = \sqrt{2} = 1.4142$ in. Ans. Hence, the largest square bar that can be planed from the round bar is 1.4142″ square.

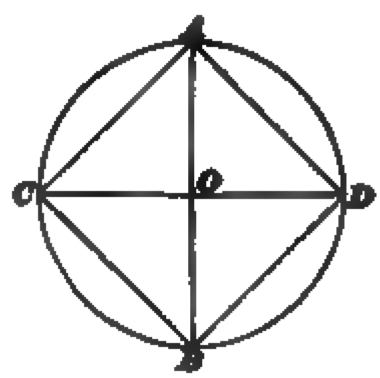

FIG. 8.

(193) The area of a circle 15 in. in diameter $= 15^2 \times .7854 = 176.715$ sq. in. Hence, the area of a sector of this circle, whose angle is $12\frac{1}{2}°$, $= 176.715 \times \frac{12\frac{1}{2}}{360} = \frac{2,208.937}{360} =$ 6.1359 sq. in. Ans. (See rule **60**, Art. **407**.)

(194) (*a*) The side of a square whose area = 103.8691 sq. in. $= \sqrt{103.8691} = 10.1916$ in. Ans.

(*b*) By rule **59**, Art. **406**, the diameter of a circle having the same area $= \sqrt{\frac{103.8691}{.7854}} = 11\frac{1}{2}$ in. Ans.

(*c*) Perimeter of the square $= 10.1916 \times 4 = 40.7664$ in.; circumference of the circle $= 11.5 \times 3.1416 = 36.1284$ in.; difference $= 40.7664 - 36.1284 = 4.638$ in. Ans.

(195) With 5 in. allowed for lap, the length of the plate which forms the shell is $46 - 5 = 41$ in. This is the circumference of the shell. By rule **56**, Art. **403**, the diameter corresponding to this circumference is $\frac{41}{3.1416} =$ 13.05 in. Ans.

(196) The convex area is the area of the outside surface, not including the area of the ends. The circumference of the base $= 26 \times 3.1416 = 81.6816$ in. This reduced to feet, since the area is to be in feet, $= 81.6816 \div 12 = 6.8068$. Using rule **62**, Art. **416**, convex area $= 6.8068 \times 10\frac{1}{2} =$ 71.4714 sq. ft. Ans.

(197) The perimeter of the base $= 4 \times 6 = 24$ in. $= 2$ ft. Convex area $= 2 \times 12 = 24$ sq. ft. The area of the bases is found as follows: In Fig. 9, $AB = 4$ in. and $AC = 2$ in.; since this is a regular hexagon, $AO = AB = 4$ in. By rule **50**, Art. **385**, $OC = \sqrt{4^2 - 2^2} = \sqrt{12} = 3.4641$ in.; area of triangle $AOB = \frac{4 \times 3.4641}{2} = 6.9282$ sq. in.; area of base $= 6.9282 \times 6 = 41.5692$, and the area of both bases $= 41.5692 \times 2 = 83.1384$ sq. in. This reduced to square feet $= \frac{83.1384}{144} = .5774$. Hence, the area of the entire surface of the column is $24 + .5774 = 24.5774$ sq. ft. Ans.

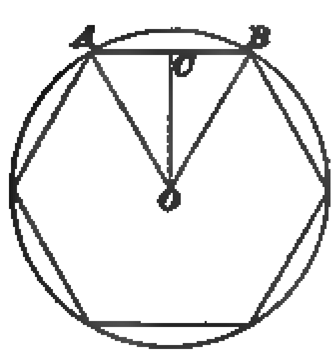

FIG. 9.

(198) The cubical contents in cubic inches = area of base in square inches $\times$ altitude in inches. The area of the base in the last example was found to be 41.5692 sq. in.;

altitude $= 12 \times 12 = 144$ in. Hence, the cubical contents $= 41.5692 \times 144 = 5{,}985.9648$ cu. in. Ans.

(199) This example is solved by combining the rules for the circular ring (see example, Art. **406**) and for the cylinder. To obtain the area of one end of the tube, we have $4^2 \times .7854 = 12.5664 =$ area of a circle 4 inches in diameter; $3.73^2 \times .7854 = 10.9272 =$ area of a circle 3.73 inches in diameter; difference $= 12.5664 - 10.9272 = 1.6392 =$ area of one end of the tube. The cubical contents $= 1.6392 \times 12 = 19.6704$ cu. in.; the weight $= 19.6704 \times .28 = 5.5$, or $5\frac{1}{2}$ lb. Ans.

(200) This example is done exactly like the one in Art. **417**, and the solution is given here without explanation.

(*a*) In the formula of rule **61**, Art. **408**, $\frac{4h^2}{3}\sqrt{\frac{D}{h} - .608}$, h in this case $= 18$, and $D = 60$.

Substituting, area $=$

$$\frac{4 \times 18^2}{3}\sqrt{\frac{60}{18} - .608} = \frac{4 \times 324}{3}\sqrt{3.333 - .608} = 432 \times \sqrt{2.725} =$$

$432 \times 1.65 = 712.8$ sq. in. This reduced to square feet $= 712.8 \div 144 = 4.95$. Hence, the steam space $= 4.95 \times 16 = 79.2$ cu. ft. Ans.

(*b*) Total area of one end of boiler in square inches $= 60^2 \times .7854 = 2{,}827.44$. From this is to be subtracted the area of the tube ends and of the segment found above.

Area of ends of tubes	$= 3.5^2 \times .7854 \times 64 =$	615.75 sq. in.
Area of segment	$=$	712.8 sq. in.
		1,328.55 sq. in.

Area of water space $= 2{,}827.44 - 1{,}328.55 = 1{,}498.89$ sq. in.

Contents of water space $= 1{,}498.89 \times 16 \times 12 = 287{,}786.88$ cu. in., and $287{,}786.88 \div 231 = 1{,}245.83$, number of gallons, or say 1,246 gal. Ans.

(201) (*a*) Area of piston $= 19^2 \times .7854 = 283.529$ sq. in., or 1.9689 square feet (rule **58**, Art. **405**).

Length of stroke plus the clearance $= 1.14 \times 2$ ft. (24 in. $= 2$ ft.) $= 2.28$ ft.

$1.9689 \times 2.28 = 4.489$ cubic feet, or the volume of steam in the small cylinder (rule **63**, Art. **417**).

(*b*) Area of piston $= 31^2 \times .7854 = 754.7694$ sq. in., or 5.2414 square feet.

Length of stroke plus the clearance $= 1.08 \times 2 = 2.16$ ft.

$5.2414 \times 2.16 = 11.321$ cubic feet, or the volume of steam in the large cylinder. Ans.

(*c*) Ratio $= \frac{11.321}{4.489}$ or $2.522 : 1$. Ans.

(202) Let the equilateral triangle ABC, Fig. 10, represent the base of the pyramid. By rule **50**, Art. **385**, the altitude AD of the triangle $= \sqrt{10^2 - 5^2} = \sqrt{75} = 8.6602$ in., and according to rule **51**, Art. **386**, the area of the triangle $= \frac{10 \times 8.6602}{2} = 43.301$ sq. in.

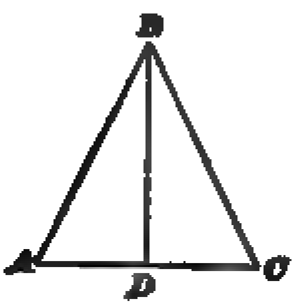

FIG. 10.

Using rule **65**, Art. **423**, volume of pyramid $=$ area of base $\times \frac{1}{3}$ altitude $= \frac{43.301 \times 10}{3} = 144.336$ cu. in. Ans.

(203) In Fig. 11, let OH be the altitude and OE the slant height of the pyramid. Connect points H and E, forming the right-angled triangle OHE, in which we have to find OH. Since it is a right pyramid, point H will fall at the center of the base $ABCD$, and, hence, the line $HE = \frac{1}{2}AB$, or 8 in.; $OE = 25$ in.; by rule **50**, Art. **385**, $OH = \sqrt{25^2 - 8^2} = \sqrt{561} = 23.6854$ in. Ans.

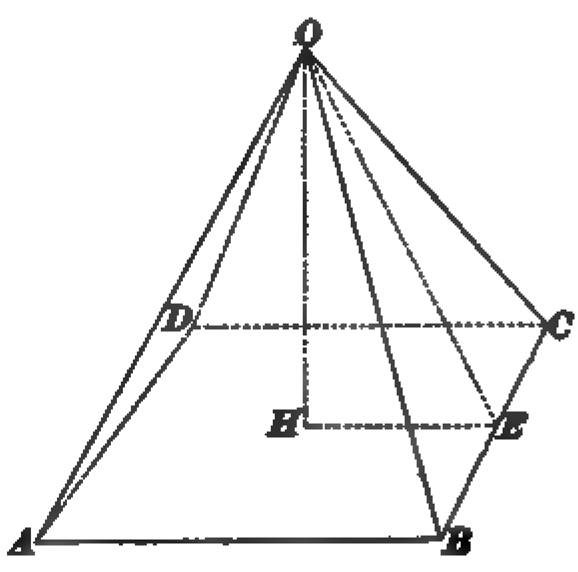

FIG. 11.

(204) The area of the convex surface = circumference of base $\times \frac{1}{2}$ slant height $= 18.8496 \times \frac{10}{2} = 94.248$ sq. in. (See rule **64**, Art. **422.**) The area of the entire surface $= 94.248$ sq. in. + the area of the base. The diameter of the base $= \frac{18.8496}{3.1416} = 6$ in.; hence, the area of the base $= 6^2 \times .7854 = 28.2744$ (rules **56** and **58**, Arts. **403** and **405**); therefore, the area of the entire surface $= 94.248 + 28.2744 = 122.5224$ sq. in. Ans.

(205) Using rule **65**, Art. **423**, volume = area of base $\times \frac{1}{3}$ altitude $= 28.2744 \times \frac{9}{3} = 84.8232$ cu. in. Ans.

(206) The vat has the form of an inverted frustum of a pyramid. Area of larger base $= 15^2 = 225$ sq. ft.; area of smaller base $= 12^2 = 144$ sq. ft. Hence, by rule **67**, Art. **427**, the contents of the vat in cubic feet $= (225 + 144 + \sqrt{225 \times 144}) \frac{11}{3} = (369 + 180) \times \frac{11}{3} = 549 \times \frac{11}{3} = 2{,}013$ cu. ft. This should be reduced to cubic inches by multiplying by 1,728, the number of cubic inches in a cubic foot. $2{,}013 \times 1{,}728 = 3{,}478{,}464$ cu. in. Since there are 231 cubic inches in a gallon, the number of gallons that the vat will hold $= \frac{3{,}478{,}464}{231} = 15{,}058.29$. Ans.

(207) The pail is in the form of a frustum of a cone. Area of larger base $= 12^2 \times .7854 = 113.0976$ sq. in. Area of smaller base $= 63.6174$ sq. in. Hence, the contents in cubic inches =

$$(113.0976 + 63.6174 + \sqrt{113.0976 \times 63.6174}) \times \frac{11}{3} =$$

$$(176.715 + \sqrt{7{,}194.9753}) \frac{11}{3} = (176.715 + 84.8232) \times \frac{11}{3} =$$

$$261.5382 \times \frac{11}{3} = 958.9734.$$

The contents of the vat in cubic inches were found in the

last example to be 3,478,464. Hence, the number of pails of water required to fill the vat $= 3{,}478{,}464 \div 958.9734 = 3{,}627.28$. Ans.

(208) By rule **66, Art. 426,** the area of the convex surface = half the sum of the perimeters of the upper and lower bases $\times$ the slant height, or $\frac{48 + 36}{2} \times 32 = 42 \times 32 = 1{,}344$ sq. in.

(209) See note following the question.

Outside diameter of upper base $= \frac{170.5}{3.1416} =$ about 54.27 in.; inside diameter $= 54.27 - 2.5 = 51.77$ in.; area of upper base $= 51.77^2 \times .7854 = 2{,}105$ sq. in., nearly.

Outside diameter of lower base $= \frac{190}{3.1416} = 60.48$ in., nearly; inside diameter $= 60.48 - 2.5 = 57.98$ in.; area of lower base $= 57.98^2 \times .7854 = 2{,}640$ sq. in., nearly.

Apply rule **67, Art. 427.** Contents of the tank in cubic inches $= (2{,}105 + 2{,}640 + \sqrt{2{,}105 \times 2{,}640}) \times \frac{7 \times 12}{3} = (4{,}745 + 2{,}357) \times 28 = 7{,}102 \times 28 = 198{,}856$ cu. in. Hence, the number of gallons $= 198{,}856 \div 231 = 860.8$. Therefore, in round numbers, the tank will hold 861 gallons.

(210) (*a*) By rule **68, Art. 429,** area of the surface $= 22.5^2 \times 3.1416 = 506.25 \times 3.1416 = 1{,}590.435$ sq. in. Ans.

(*b*) Using rule **69, Art. 430,** the cubical contents = the cube of the diameter $\times .5236 = 11{,}390.625 \times .5236 = 5{,}964.1313$ cu. in. Ans.

(211) Having given the area of the surface, to find the volume we must first obtain the diameter. The process is just the reverse of finding the surface when the diameter is given. Hence, the diameter $= \sqrt{201.0624 \div 3.1416} = \sqrt{64} = 8$ in. By rule **69, Art. 430,** volume $= 8^3 \times .5236 = 268.0832$ cu. in. Ans.

(212) (*a*) Given $OB = \frac{16}{2}$, or 8 inches, and $OA = \frac{13}{2}$, or $6\frac{1}{2}$ inches, to find the volume, area, and weight (see Fig. 12):

Radius of center circle equals $\frac{8 + 6.5}{2}$, or $7\frac{1}{4}$ inches.

Length of center line $= 2 \times 3.1416 \times 7\frac{1}{4} = 45.5532$ inches.

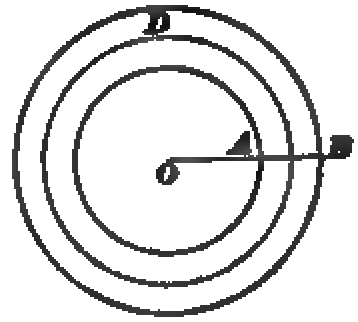

FIG. 12.

The radius of the inner circle is $6\frac{1}{2}$ inches, and of the outer circle 8 inches; therefore, the diameter of the cross-section on the line AB is $1\frac{1}{2}$ inches. Then, according to rule **70**, Art. **431**, the area of the ring is $1\frac{1}{2} \times 3.1416 \times 45.553 = 214.665$ square inches. Ans.

Diameter of cross-section of ring $= 1\frac{1}{2}$ inches.

Area of cross-section of ring $= (1\frac{1}{2})^2 \times .7854 = 1.76715$ sq. in. Ans.

By rule **71**, Art. **432**, volume of ring $= 1.76715 \times 45.553 = 80.499$ cu. in. Ans.

(*b*) Weight of ring $= 80.499 \times .261 = 21$ lb. Ans.

(213) The volume of the ring equals the product of its cross-sectional area and its length on line D; therefore, cross-sectional area $= \frac{144.349}{20.42} = 7.069$ sq. in.

The diameter $AB = \sqrt{\frac{7.069}{.7854}} = 3$ in.

Area of surface = circumference of circle, of which AB is diameter, $\times$ length $= 3.1416 \times 3 \times 20.42 = 192.45$ square inches. Ans.

MECHANICS.

(QUESTIONS 214-312.)

(214) See Arts. **435** and **436.**

(215) Reducing 14 minutes to seconds, $14 \times 60 = 840$ seconds.

$$840 \times 40 = 33{,}600 \text{ ft.} = 6\frac{4}{11} \text{ miles. Ans.}$$

(216) See Art. **477.**

(217) $W \times Fb = P \times Fc$, or $W \times 3\frac{1}{2} = 21 \times 85$.

$$21 \times 85 = 1{,}785. \quad 1{,}785 \div 3.5 = 510 \text{ lb. Ans.}$$

(218) Applying rule **84**, Art. **499**,

$$\frac{80 \times 28}{21} = 106\frac{2}{3} \text{ revolutions per minute. Ans.}$$

(219) (*a*) Applying rule **86**, Art. **508**,

$$\text{pitch diameter} = \frac{1\frac{1}{2} \times 50}{3.1416} = 23.87''. \quad \text{Ans.}$$

(*b*) See Art. **507.** Addendum $= .3$ of the pitch. $1.5 \times .3 = .45''$. $.45 \times 2 = .9''$ = difference between the diameter of the pitch circle and the outside diameter. Hence, outside diameter $= 23.87 + .9 = 24.77''$. Ans.

(220) Apply rule **91**, Art. **511.**

$$n = \frac{212 \times 45}{180} = 53 \text{ R. P. M. Ans.}$$

(221) Apply rule **96**, Art. **521.**

$$W = \frac{6.2832 \times 24 \times 11}{\frac{1}{13}} = 21{,}563.94 \text{ lb. Ans.}$$

(222) The pull on the support equals the centrifugal force of the ball. Hence, applying rule **98**, Art. **535**, the centrifugal force =

$$.00034 \times 5 \times \frac{32}{12} \times 350^2 = 555\frac{1}{3} \text{ lb. Ans.}$$

(223) Apply rule **100**, Art. **545**.

$$\text{Kinetic energy} = \frac{2 \times 600^2}{64.32} = 11,194 \text{ ft.-lb. Ans.}$$

(224) 7 ft. = 84 in.; arc of contact $= \frac{84}{63 \times 3.1416} \times 360° = 153°$. $800 + 3\,(180 - 153) = 881$. Apply rule **102**, Art. **552**, using 881 instead of 800, as explained in Art. **554**.

$$W = \frac{881 \times 150}{3,000} = 44.05''.$$

The width of a double belt is two-thirds of this, or $44.05 \times \frac{2}{3} = 29.37''$, or say $29.5''$. Ans.

(225) See Arts. **437** to **461**.

(226) There are 1,760 yd. in 1 mile. If a man can run 100 yd. in 12 seconds, in 1 second he can run $\frac{100}{12}$ yd.; that is, his velocity is $\frac{100}{12}$ yd. per sec. Applying rule **74**, Art. **468**,

$$1,760 \div \frac{100}{12} = 1,760 \times \frac{12}{100} = 211.2 \text{ sec.} = 3 \text{ min. } 31.2 \text{ sec. Ans.}$$

(227) See Art. **478**.

(228) See Arts. **493** and **494**.

(229) Applying rule **84**, Art. **499**,

$$\text{speed of engine} = \frac{91 \times 108}{13 \times 12} = 63 \text{ revolutions per minute. Ans}$$

(230) Apply rule **86**, Art. **508**.

$$\text{Pitch diameter} = \frac{2\frac{1}{2} \times 192}{3.1416} = 152.79''. \text{ Ans.}$$

(231) Apply rule **92**, Art. **511.**

Speed of driving gear $= \frac{81 \times 80}{18} = 360$ revolutions per minute. Ans.

(232) Apply rule **96**, Art. **521.**

$$W = \frac{6.2832 \times 60 \times 26}{\frac{1}{8}} = 78{,}414.336 =$$

the theoretical pressure. Since the efficiency is but 40%, the actual pressure is $78{,}414.336 \times .40 = 31{,}365.7$ lb. Ans.

(233) See Art. **537.**

(234) First determine the speed of the center of gravity of the section in feet per second. This point revolves in a circle whose diameter is 6 ft. $1\frac{3}{4}'' \times 2 = 12$ ft. $3\frac{1}{2}'' = 12.2917$ ft. Distance traveled in one revolution $= 12.2917 \times 3.1416 = 38.6156$ ft. Distance traveled in one second $= \frac{38.6156 \times 150}{60} = 96.539$ ft. Hence, applying rule **100**, Art. **545**, kinetic energy $= \frac{13{,}000 \times 96.539^2}{64.32} = 1{,}883{,}661.7$ ft.-lb. Ans.

(235) Arc of contact $= \frac{18}{14 \times 3.1416} \times 360° = 147°$.

$$800 + 3\,(180 - 147) = 899.$$

Applying rule **102**, Art. **552**,

$$W = \frac{899 \times 2.5}{2{,}000} = 1.12''. \quad \text{Ans.}$$

The nearest standard width is 1 in.

(236) See Arts. **447** and **448.**

(237) $\frac{48}{12} \times 3.1416 = 12.5664$ ft. = circumference of pulley. $\frac{3{,}000}{12.5664} = 238.73$ revolutions in 1 minute, or 60 seconds. To make 100 revolutions will require $\frac{100}{238.73} \times 60 = 25.13$ sec., nearly. Ans.

(238) 4 ft. 6″ = 54″. $54 \times 2 \times \frac{3}{4} \times .261 = 21.141$ lb. = weight of lever. Considering the weight of the lever to be concentrated at its center of gravity, we have three weights of 47, 21.141, and 71 lb., with the smaller weight, $\frac{54}{2} = 27''$, from the other two. To find the center of gravity of the two large weights, apply rule **75**, Art. **478**, $\frac{47 \times 54}{71 + 47} = 21.508''$ = the distance $b\,c$ in Fig. 13. Consider both weights

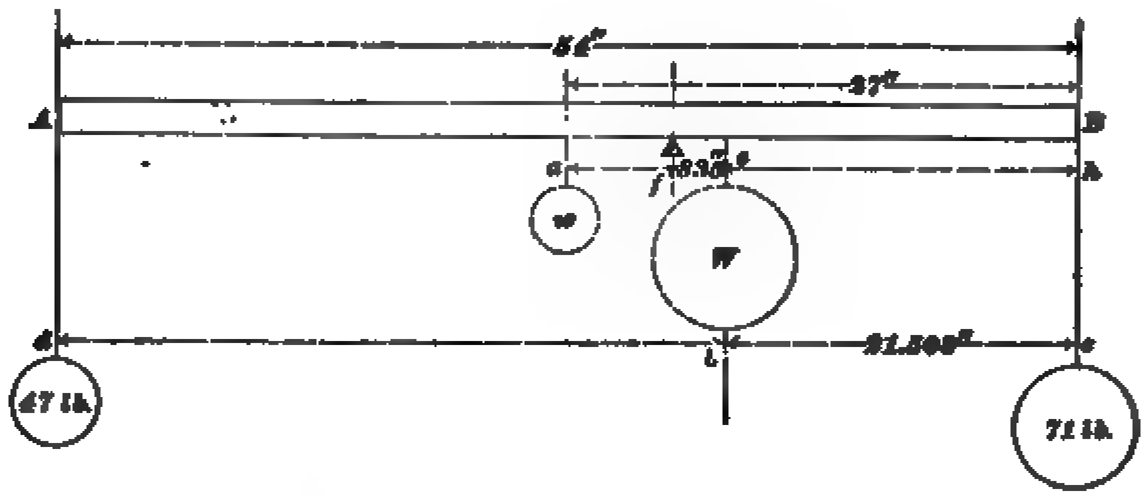

FIG. 13.

to be concentrated at b; that is, imagine both weights removed, and to be replaced by the dotted weight W, equal to $71 + 47 = 118$ lb. The dotted circle w represents the weight of the bar. The distance $a\,e = 27 - 21.508 = 5.492''$. Distance of balancing point f from e is found by means of rule **75**, Art. **478**, to be $\frac{21.141 \times 5.492}{118 + 21.141} = .834''$. Finally, $f\,h = 21.508 + .834 = 22.342''$ = short arm. Ans.

$54 - 22.342 = 31.658''$ = long arm. Ans.

(239) See Art. **495**.

(240) Apply rule **84**, Art. **499**, after reducing the 2 ft. to inches.

Revolutions of driver $= \frac{32 \times 63}{24} = 84$ per min. Ans.

(241) Apply rule **87**, Art. **508**.

Number of teeth in the gear $= \frac{11.48 \times 3.1416}{1\frac{1}{8}} = 32$. Ans.

(242) Apply rule **92**, Art. **511**, to find the number of revolutions of the driving gear; thus, $\frac{75 \times 88}{44} = 150 =$ revolutions per minute of the 8″ pulley also. Apply rule **81**, Art. **496**, to find the diameter of pulley on the shaft. $\frac{8 \times 150}{200} = 6''$. Ans.

(243) (*b*) Apply rule **96**, Art. **521**.

$$W = \frac{6.2832 \times 25 \times 15}{\frac{1}{4}} = 9{,}424.8 \text{ lb.} = \text{theoretical pressure.}$$ Ans.

(*a*) $9{,}424.8 - 5{,}000 = 4{,}424.8$ lb. Ans.

(244) See Art. **537**.

$$\frac{51}{62.5} = .816, \text{ the specific gravity.}$$ Ans.

(245) 660 ft. per min. $= \frac{660}{60} = 11$ ft. per sec.

Applying rule **100**, Art. **545**,

$$\text{kinetic energy} = \frac{325 \times 11^2}{64.32} = 611.4 \text{ ft.-lb., nearly.}$$ Ans.

(246) Applying rule **105**, Art. **557**,

$$\text{horsepower} = .01 \times 1200 \times 1^2 = 12.$$ Ans.

(247) See Arts. **462** to **464**.

(248) See Art. **469**.

(249) In Fig. 14 *A B C* represents the triangle. Find center of gravity by means of rule **78**, Art. **481**. The distance of the center of gravity from the side *A C* $= 1\frac{3}{16}''$. Ans.

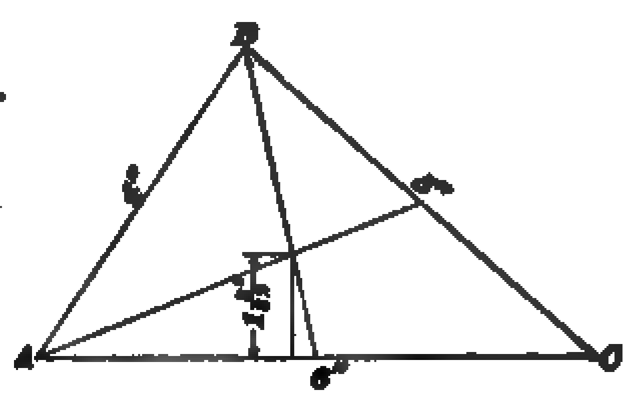

FIG. 14.

(250) Apply rule **81**, Art. **496**,

$$\text{Diameter of driver} = \frac{36 \times 60}{40} = 54''.$$ Ans.

(251) (*a*) Applying rule **83**, Art. **498**,

$$\frac{12 \times 80}{8} = 120 \text{ R. P. M.} \quad \text{Ans.}$$

(*b*) Applying rule **83**,

$$\frac{120 \times 20}{6} = 400 \text{ R. P. M.} \quad \text{Ans.}$$

(*c*) Applying rule **83** again,

$$\frac{400 \times 6}{4} = 600 \text{ R. P. M.} \quad \text{Ans.}$$

(252) Applying rule **87**, Art. **508**,

$$\frac{3.1416 \times 34.15}{1\frac{3}{8}} = 78 \text{ teeth.} \quad \text{Ans.}$$

(253) Since there are two parts to the rope, the pulleys will raise a load of $225 \times 2 = 450$ lb. Ans. (See rule **93**, Art. **516**.)

(254) 5 ft. 6″ = 66″.

(*a*) $66 \div 6 = 11$ = velocity ratio. Ans.

(*b*) $11 \times 5 = 55$ lb. Ans.

(255) See Art. **530**. $55 \times .65 = 35.75$ lb. Ans.

(256) See Art. **539**. Since one cu. ft. of platinum weighs 1,343.8 lb., 1 cu. in. weighs $\frac{1,343.8}{1,728}$ lb., and 10 lb. will contain $10 \div \frac{1,343.8}{1,728} = 10 \times \frac{1,728}{1,343.8} = 12.86$ cu. in., nearly. Ans.

(257) See Arts. **549** and **550**.

(258) Speed of a point on the pitch circle in feet per minute $= \frac{30}{12} \times 3.1416 \times 100 = 785.4$ ft. per min. Applying rule **105**, Art. **557**,

Horsepower $= .01 \times 785.4 \times 1.57 \times 1.57 = 19.36$. Ans.

(259) See Art. **464**.

(260) See Art. **469**.

(261) Volume of sphere $= .5236 \times 5^3 = 65.45$ cu. in. 1 cu. in. of cast iron weighs .261 lb.; hence, weight of ball =

$65.45 \times .261 = 17.08$ lb. Weight of a cu. in. of steel is .284 lb.; hence, weight of handle $= \left(\frac{7}{8}\right)^2 \times .7854 \times 40 \times .284 =$ 6.83 lb., distance of center of gravity of rod from center of ball $= \frac{40}{2} + \frac{5}{2} = 22\frac{1}{2}''$. Apply rule **75**, Art. **478.** Distance of center of gravity of both ball and rod from center of ball $=$ $\frac{6.83 \times 22.5}{17.08 + 6.83} = 6.427''$. Ans.

(262) Applying rule **81**, Art. **496.**

$$\text{diameter of driver} = \frac{180 \times 30}{240} = 22\frac{1}{2}''. \quad \text{Ans.}$$

(263) Apply rule **85**, Art. **501**,

$$P = \frac{6{,}000 \times 6 \times 5 \times 8 \times 3}{18 \times 12 \times 15 \times 12} = 111\frac{1}{9} \text{ lb.}$$

Since there is a loss of 20%, $111\frac{1}{9}$ represents 80% of the total force. Hence, the force actually required $= 111\frac{1}{9} \div .80 =$ $138\frac{8}{9}$ lb. Ans.

(264) Apply rule **88**, Art. **508.**

$$\text{Pitch} = \frac{3.1416 \times 24.16}{38} = 1.997''. \quad \text{Ans.}$$

(265) (See Art. **516.**) Since there are eight parts of the rope, the force required $= 1{,}890 \div 8 = 236\frac{1}{4}$ lb. Ans.

(266) (*a*) (See Art. **523.**) Velocity ratio $= \frac{1{,}000}{50} = 20$ Ans.

(*b*) (See Arts. **529** and **530.**) Efficiency $= \frac{50}{95} = .5263 =$ 52.63%. Ans.

(267) Volume $= \left(\frac{1}{2}\right)^2 \times .7854 \times 10 = 1.963$ cu. in. One cu. in. of lead weighs .411 lb. (see table of Specific Gravities); consequently, $1.963 \times .411 = .807$ lb. $= 12.91$ oz. Ans.

(268) Apply rule **101**, Art. **551**,

Length of belt $= 3\frac{1}{4} \times \frac{11+7}{2} + 2 \times 38 = 105\frac{1}{4}$ ft. $=$ 105 ft. 3 in. Ans.

(269) (*a*) $18 \times 60 \times 60 = 64{,}800$ miles per hour. Ans.

(*b*) $64{,}800 \times 24 = 1{,}555{,}200$ miles per day. Ans.

(270) See Art. **463.**

(271) See Arts. **469** and **470.**

(272) Length of power arm $= 4$ ft. $- 4'' = 48'' - 4'' = 44''$. Hence, $1{,}500 \times 4 = 6{,}000$. $x \times 44 = 6{,}000$, or $x = \frac{6{,}000}{44} = 136\frac{4}{11}$ lb. Ans. (See Art. **487.**)

(273) Length of power arm $= 4$ ft. $= 48''$. Hence, $1{,}500 \times 4 = 6{,}000$. $x \times 48 = 6{,}000$, or $x = \frac{6{,}000}{48} = 125$ lb. Ans.

(274) Apply rule **82**, Art. **497**,

$$\text{Diameter of pulley} = \frac{10 \times 88}{110} = 8 \text{ ft.} \quad \text{Ans.}$$

(275) See Arts. **503-505.**

(276) Apply rule **88**, Art. **508**,

$$\text{Pitch} = \frac{3.1416 \times 36.56}{42} = 2.735''. \quad \text{Ans.}$$

(277) (See Art. **518.**) According to rule **94**,

$$P = \frac{Wh}{l} = \frac{4{,}000 \times 45}{400} = 450 \text{ lb.} \quad \text{Ans.}$$

(278) See Arts. **523** and **530.**

(279) One foot of the wire will weigh $\left(\frac{1}{16}\right)^2 \times .7854 \times 12 \times .303 = .011155$ lb. (See Art. **539**; also table of Specific Gravity of Metals.) Consequently, 10 lb. will contain $\frac{10}{.011155} = 896$ ft., nearly. Ans.

(280) 14 ft. = 168″. Applying rule **101**, Art. **551**, length of the belt $= 3\frac{1}{4} \times \frac{18+14}{2} + 2 \times 168 = 388'' = 32$ ft. 4″. Ans.

(281) 30 miles per hour $= 5{,}280 \times 30 = 158{,}400$ ft. per hour $= \frac{158{,}400}{60} = 2{,}640$ ft. per min. $= \frac{2{,}640}{60} = 44$ ft. per sec. Ans.

(282) $\frac{28}{12} \times 1{,}500 = 3{,}500$ ft. in 6 min. $= \frac{3{,}500}{360} = 9\frac{13}{18}$ ft. per sec. (since 6 min. $= 6 \times 60 = 360$ sec.) Ans.

(283) See Art. **471.**

(284) Apply rule **82**, Art. **497.**

$$\text{Diameter} = \frac{40 \times 120}{160} = 30''. \quad \text{Ans.}$$

(285) See Arts. **506** and **507.**

(286) See Arts. **509** and **510.**

(287) The weight which comes on the block and tackle is the same as the force required to pull the body up the plane, or is equal to $\frac{50{,}000 \times 125}{1{,}200} = 5{,}208\frac{1}{3}$ lb. (See rule **94**, Art. **518.**) Since there are 12 parts to the rope, the force required to be exerted on the free end is, according to rule **93**, Art. **516**, $5{,}208 \div 12 = 434$ lb. Ans.

(288) See Arts. **533** and **534.**

(289) See Arts. **540, 543, 545,** and **547.**

(290) 19 ft. 3″ = 231″. Applying rule **101**, Art. **551**, length of belt $= 3\frac{1}{4} \times \frac{20+8}{2} + 2 \times 231 = 507\frac{1}{2}'' = 42$ ft. $3\frac{1}{2}''$. Ans.

(291) (*a*) 15 miles per hour $= \frac{15 \times 5{,}280}{60 \times 60} = 22$ ft. per sec. Since the bodies are moving in opposite directions, they are moving *away* from each other, and their distance apart is increasing at the constant rate of $11 + 22 = 33$ ft.

per sec. In 8 min. the distance between them will be $\frac{33 \times 8 \times 60}{5,280} = 3$ miles. Ans.

(*b*) $825 \div 33 = 25$ sec. Ans.

(292) 2 min. 10 sec. = 130 sec. 2 miles = 10,560 ft. Applying rule **72**, Art. **465**, velocity $= \frac{10,560}{130} = 81.23$ ft. per sec. Ans.

(293) See Art. **476.**

(294) Apply rule **80**, Art. **490**, letting x represent the required force.

$$x \times 30 \times 20 \times 10 \times 15 = 1,250 \times 6 \times 5 \times 4 \times 7, \text{ or}$$

$$x = \frac{1,250 \times 6 \times 5 \times 4 \times 7}{30 \times 20 \times 10 \times 15} = 11\tfrac{2}{3} \text{ lb.} \quad \text{Ans.}$$

(295) Applying rule **83**, Art. **498**,

$$n = \frac{20 \times 150}{16} = 187\tfrac{1}{2} \text{ revolutions per minute.} \quad \text{Ans}$$

(296) See Art. **507.**

(297) Apply rule **89**, Art. **511.**

$$\text{Number of teeth} = \frac{60 \times 40}{100} = 24. \quad \text{Ans.}$$

(298) See rule **95**, Art. **518.**

$$P = \frac{750 \times 50}{80} = 468\tfrac{3}{4} \text{ lb.} \quad \text{Ans.}$$

(299) Apply rule **98**, Art. **535.**

Centrifugal force $= .00034 \times 128 \times \frac{8\frac{1}{4}}{12} \times 180^2 = 1,028.16$ lb. Ans.

(300) One cubic foot of water weighs 62.5 lb.; hence, 20 cu. ft. weigh $62.5 \times 20 = 1,250$ lb. The work done = $1,250 \times 50 = 62,500$ ft.-lb. Ans.

(301) Arc of contact $= \frac{21}{15 \times 3.1416} \times 360° = 160°$.

$800 + 3(180 - 160) = 860$. Applying rule **103**, Art. **553**,

$$H = \frac{5 \times 1,960}{860} = 11.4 \text{ H. P.} \quad \text{Ans.}$$

(302) $18{,}000 + 10{,}000 = 28{,}000$ lb. = the load which the screw must overcome. Applying rule **96**, Art. **521**,

$$P = \frac{28{,}000 \times \frac{1}{3}}{6.2832 \times 15} = 99 \text{ lb., nearly.} \quad \text{Ans.}$$

(303) $\frac{9 \times 3.1416 \times 100 \times 60}{5{,}280} = 32.13$ miles per hour = the velocity. Applying rule **73**, Art. **467**,

$$32.13 \times 1\frac{1}{4} = 40.16\frac{1}{4} \text{ miles.} \quad \text{Ans.}$$

(304) If the ball fitted the gun loosely, and the gun was held horizontally, the ball would roll out and fall to the floor. since, according to the first law of motion, every body tends to preserve its velocity unless acted upon by some force The ball has a velocity due to the train of 100 ft. per sec When the gun is fired the force applied to the ball apparently gives it a velocity of 100 ft. per sec. in the opposite direction, but it really stops the ball and brings it to rest relatively to a point on the earth. The gun and car keep up their motion and draw away from the ball, which is stationary with respect to a point on the earth, and the ball falls to the ground.

(305) Radius of drum $= 5 \div 2 = 2\frac{1}{2}$ in.; radius of circle described by handle $= 14\frac{1}{2}$ in.

Velocity ratio $= \frac{14.5}{2.5} = 5.8$.

Weight = power × velocity ratio $= 30 \times 5.8 = 174$ lb. Ans.

(306) Apply rule **83**, Art. **498**.

Revolutions per minute of the countershaft $= \frac{42 \times 108}{36} = 126$. Ans.

(307) See Art. **507**.

(308) Apply rule **90**, Art. **511**.

$$\frac{34 \times 360}{170} = 72 \text{ teeth.} \quad \text{Ans.}$$

(309) Answer from your own observation.

(310) 9 ft. $10\frac{3}{4}'' = 9.9$ ft., very nearly. $9.9 \div 2 = 4.95$ ft. = the radius. 9 ft. $10\frac{3}{4}'' = 118.75''$.

Weight of rim $= 118.75 \times 3.1416 \times 22 \times 2\frac{1}{2} \times .261 =$ 5,355 lb., nearly.

Applying rule **98**, Art. **535**,

$F = .00034 \times 5{,}355 \times 4.95 \times 210^2 = 397{,}450$ lb. Ans.

(311) In this case the force acting on the piston is $10^2 \times .7854 \times 41.38$; the distance through which the force acts or the distance the piston moves in one minute is $\frac{16}{12} \times 450$. Therefore, the number of foot-pounds of work done in 1 minute is $10^2 \times .7854 \times 41.38 \times \frac{16}{12} \times 450 = 1{,}949{,}991$ ft.-lb.

Dividing by 33,000 to obtain the horsepower,

$$\frac{1{,}949{,}991}{33{,}000} = 59{,}091 \text{ horsepower, nearly. Ans.}$$

(312) Since the width of a double belt is but $\frac{2}{3}$ of that of a single belt to transmit the same horsepower, a single belt doing the same work as the $20''$ double belt, in this example, must be $20 \div \frac{2}{3} = 20 \times \frac{3}{2} = 30''$ wide.

Arc of contact $= \frac{5.75}{4 \times 3.1416} \times 360° = 165°$. $800 + 3\,(180 - 165) = 845$.

Applying rule **103**, Art. **553**,

$$H = \frac{30 \times 2{,}800}{845} = 99.4 \text{ H. P. Ans.}$$

MECHANICS.

(QUESTIONS 313–333.)

(313) See Art. **563.**

(314) Fig. 15 shows a sketch of the apparatus. The load which can be lifted is evidently equal to the area of the piston multiplied by the pressure in pounds per sq. in., or $19^2 \times .7854 \times 90 = 25{,}517.6$ lb. Ans.

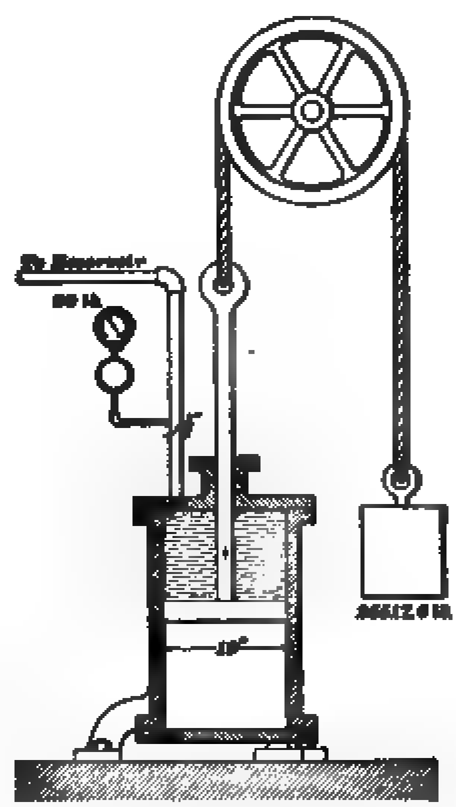

FIG. 15.

(315) See Art. **593.**

(316) See Art. **623.**

(317) Applying rule **115,** Art. **628,**

$$12{,}000 \times \left(\frac{3}{8}\right)^2 = 1{,}687.5 \text{ lb. Ans.}$$

(318) Apply rule **120,** Art. **633.**

$$\text{Load} = 1{,}000 \times \left(5\frac{1}{4}\right)^2 = 27{,}562.5 \text{ lb. Ans.}$$

(319) Applying rule **126,** Art. **649,**

$$\text{force} = 6^2 \times .7854 \times 60{,}000 = 1{,}696{,}464 \text{ lb. Ans.}$$

(320) Apply rule **129,** Art. **655.**

Diameter $= \sqrt[3]{\frac{70 \times 40}{116}} = 2.89''$. A $2\frac{15}{16}''$ shaft would be used, that is, one 3" diam. before turning down. Ans.

(321) The pressure per square inch on $c = 24 \div .6 = 40$ lb.

$$\left.\begin{array}{l}\text{Then, pressure on } d = 3 \times 40 = 120 \text{ lb.}\\ \text{pressure on } e = 6 \times 40 = 240 \text{ lb.}\\ \text{pressure on } f = 2 \times 40 = 80 \text{ lb.}\\ \text{pressure on } a = 14 \times 40 = 560 \text{ lb.}\\ \text{pressure on } b = 9 \times 40 = 360 \text{ lb.}\end{array}\right\} \text{Ans.}$$

(322) (*a*) $36'' = 3$ ft. Lower base of cylinder is $40 + 3 = 43$ ft. $= 516''$ below the surface of the water. Applying rule **107**, Art. **570**,

upward pressure $= 20^2 \times .7854 \times 516 \times .03617 =$ 5,863.39 + lb. Ans.

(*b*) 40 ft. $= 480''$. Applying rule **106**, Art. **567**,

downward pressure $= 20^2 \times .7854 \times 480 \times .03617 = 5.454.32$ lb. Ans.

(323) See Arts. **588** and **589**.

(324) Apply rule **113**, Art. **626**.

$$\text{Stress} = \frac{12,400}{3.5} = 3,543 \text{ lb. per sq. in., nearly.} \quad \text{Ans.}$$

(325) Apply rule **116**, Art. **630**.

Safe load $= 100 \times 4^2 = 1,600$ lb. Ans.

(326) Area of cross-section $= 8^2 \times .7854 = 50.2656$ sq. in. 10 ft. $= 120'' = L$. Crushing strength $= 3.5$ (see Table 18, Art. **636**). $a = 187.5$ (see Table 21, Art. **637**). Substituting these values in the formula under rule **121**, Art. **638**,

$$W = \frac{3.5 \times 50.2656}{1 + \frac{120^2}{187.5 \times 8^2}} = 80 \text{ tons, very nearly.}$$

$$\text{Hence, } 80 \div 6 = 13\frac{1}{3} \text{ tons} = \text{safe load.} \quad \text{Ans.}$$

(327) (See Art. **649**.) Area to be sheared $= 1 \times 3.1416 \times \frac{7}{16} = 1.37445$ sq. in. Applying rule **126**, Art. **649**, force $= 1.37445 \times 40,000 = 54,978$ lb. Ans.

(**328**) Apply rule **127**, Art. **653.** (See Table 25.)

$$\text{Horsepower} = \frac{(1\tfrac{7}{8})^3 \times 180}{95} = 12.49. \quad \text{Ans.}$$

(**329**) See Art. **567.**

(**330**) $\dfrac{400}{7.5^2 \times .7854} = 9.054 +$ lb. per sq. in. Pressure on $a = 9.054 \times 2^2 \times .7854 = 28.44 +$ lb. = weight which must be laid upon a. Ans.

(**331**) Applying rule **109**, Art. **594,**

$$\text{new pressure} = \frac{42 \times 20\tfrac{1}{4}}{42 + 14} = 15.19 \text{ lb. per sq. in., nearly.}$$

Ans.

(**332**) Area of cross-section $= \left(1\dfrac{1}{2}\right)^2 \times .7854 = 1.7671$ sq. in.

Apply rule **111**, Art. **626.**

Safe steady load $= 12{,}000 \times 1.7671 = 21{,}205.2$ lb. Ans.

(**333**) Apply rule **116**, Art. **630.**

$$\text{Load} = 100 \times 6^2 = 3{,}600 \text{ lb.} \quad \text{Ans.}$$

(**334**) Substituting the values of $C = 40$, $S = 14^2 \times .7854 - 11.5^2 \times .7854 = 50.0693$, $L = 20 \times 12 = 240$, $a = 562.5$, and $d = 14$, in the formula given in connection with rule **121**, Art. **638,** we have

$$W = \frac{40 \times 50.0693}{1 + \dfrac{240^2}{562.5 \times 14^2}} = \frac{2{,}002.772}{1.5225} = 1{,}315.45 \text{ tons.}$$

$$\frac{1{,}315.45}{6} = 219.24 \text{ tons.} \quad \text{Ans.}$$

(**335**) (See Art. **649.**) Area sheared $= 1\dfrac{1}{2} \times 3.1416 \times \dfrac{3}{4} = 3.5343$ sq. in. Force $= 3.5343 \times 60{,}000 = 212{,}058$ lb.

Ans.

(**336**) Apply rule **106**, Art. **567.**

$$\text{Pressure} = 46 \times 18 \times .03617 = 29.95 \text{ lb.} \quad \text{Ans.}$$

(**337**) (See Art. **574.**)

(338) Apply rule **109**, Art. **594.**

$$\text{Pressure} = \frac{94.7 \times \frac{5}{8}}{2\frac{1}{2}} = 23.675 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(339) Area of cross-section $= 1\frac{3}{4} \times 3 = 5.25$ sq. in. Applying rule **111**, Art. **626**,

Safe load $= 5.25 \times 6{,}000 = 31{,}500$ lb. Ans.

(340) Apply rule **117**, Art. **630.**

Circumference $= .1\sqrt{W} = .1\sqrt{2{,}400} = 4.9''$. Ans.

(341) First find the total steam pressure, or load, which comes on the rod. It equals the area of the piston in square inches multiplied by the pressure per sq. in. Representing the area by A, the load W must equal $A \times 100 = 100\,A$. Applying rule **121**, Art. **638**, $a = 1{,}500$, $C = 18$, $L = 4$ ft. $8'' = 56''$, $S = 3.5^2 \times .7854 = 9.6211$, and $d = 3.5$; hence,

$$W = 100\,A = \frac{18 \times 9.6211}{1 + \frac{56^2}{1{,}500 \times 3.5^2}} = \frac{173.1798}{1.1707} = 147.9284 \text{ tons.}$$

Dividing by 6 to obtain the safe pressure, $\frac{147.9284}{6} = 24.65473$ tons $= 49{,}309.46$ lb.

Hence, $W = 100\,A = 49{,}309.46$ lb. If $100\,A = 49{,}309.46$, $A = \frac{49{,}309.46}{100} = 493.0946$ sq. in. = area of piston, and diameter $= \sqrt{\frac{493.0946}{.7854}} = 25''$, nearly. Ans.

(342) See Art. **650.**

(343) 8 ft. $7'' = 103''$. If the tank contained water instead of oil, the pressure would be $\left(\frac{3}{8}\right)^2 \times .7854 \times 103 \times .03617$ lb., and the pressure per sq. in. would be

$$\frac{(\frac{3}{8})^2 \times .7854 \times 103 \times .03617}{(\frac{3}{8})^2 \times .7854} = 103 \times .03617 = 3.72551 \text{ lb.}$$

Multiplying by the specific gravity of the oil, $3.72551 \times .92 = 3.427 +$ lb. per sq. in. Ans.

(344) The pressure per sq. in. exerted by the plunger C is $\frac{160}{(\frac{5}{8})^2 \times .7854}$ lb. Consequently, the pressure exerted on the bottom of the plunger a is $12^2 \times .7854 \times \frac{160}{(\frac{5}{8})^2 \times .7854} =$ 58,982.4 lb. Hence, weight which can be raised = 58,982.4 − 600 = 58,382.4 lb. Ans.

(345) Apply rule **109**, Art. **594**.

$$\text{Pressure} = \frac{1.11 \times 18}{.3} = 66.6 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(346) Total pressure on the head $= 19^2 \times .7854 \times 180 =$ 51,035 lb. Tension in each stud $= \frac{51,035}{14} = 3,645$ lb. Applying rule **112**, Art. **626**,

$$\text{area} = \frac{3,645}{5,000} = .729 \text{ sq. in.} \quad \text{Ans.}$$

(347) Apply rule **117**, Art. **630**.

$$\text{Circumference} = .1\sqrt{4,200} = 6.48'', \text{ say } 6\frac{1}{2}''. \quad \text{Ans.}$$

(348) Substitute in the formula given in connection with rule **121**, Art. **638**. For this case, $C = 18$, $S = 6 \times 2\frac{1}{2} = 15$ sq. in., $L = 10 \times 12 = 120''$, $a = 1,500$, and $d = 2\frac{1}{2}$. Consequently,

$$W = \frac{18 \times 15}{1 + \frac{120^2}{1,500 \times 2.5^2}} = 106.467 \text{ tons.}$$

$$\frac{106.467}{6} = 17.7445 \text{ tons} = 35,489 \text{ lb.} \quad \text{Ans.}$$

(349) $2\frac{7}{16}''$ shafting. See Art. **650**.

(350) (*a*) 8 ft. = 96″. $96^2 \times .7854 \times .03617 \times 10 = 2,618$ lb. Ans.

(*b*) 6 ft. = 72″. $72^2 \times .7854 \times .03617 \times 10 = 1,473$ lb., nearly. Ans.

(351) This example may be solved in two ways; first, by finding the pressure on each side and adding the results, or, second, by multiplying the area of the cube in sq. in. by the depth of its center of gravity in inches and by .03617. Area $= 2 \times 2 \times 144 \times 6 = 3{,}456$ sq. in. Depth of center of gravity $= (50 + 1) \times 12 = 612''$. Then, total pressure $= 3{,}456 \times 612 \times .03617 = 76{,}502 +$ lb. Ans.

(352) Apply rule **110**, Art. **595.**

$$\text{New volume} = \frac{45 \times 25}{15} = 75 \text{ cu. ft.}$$

Hence, $75 - 25 = 50$ cu. ft. = volume of second vessel. Ans.

(353) Total pressure against the head $= 44^2 \times .7854 \times 100 = 152{,}053.44$ lb. Applying rule **112**, Art. **626**,

$$\text{area of studs} = \frac{152{,}053.44}{5{,}000} = 30.41 \text{ sq. in., nearly.}$$

$$30.41 \div 1.057 = 29 \text{ studs.} \quad \text{Ans.}$$

(354) Apply rule **118**, Art. **633.**

$$\text{Load} = 600 \times 4^2 = 9{,}600 \text{ lb.} \quad \text{Ans.}$$

(355) Apply rule **122**, Art. **642.**

$$\text{Load} = \frac{2.5^2 \times 1.5 \times 100}{4\frac{4}{12}} = 201 \text{ lb., nearly.} \quad \text{Ans.}$$

(356) Apply rule **127**, Art. **653.** (See Table 25.)

$$\text{Horsepower} = \frac{(2\frac{1}{16})^3 \times 120}{85} = 20.445. \quad \text{Ans.}$$

(357) 5 ft. $= 60''$. $60 + 1\frac{1}{2} = 61\frac{1}{2}''$ = distance from bottom of board to surface of the water. Apply rule **107**, Art. **570.**

$$8 \times 20 \times 61\frac{1}{2} \times .03617 = 355.91 + \text{ lb.} \quad \text{Ans.}$$

(358) Yes. Because the difference between the upward and downward pressures upon the body, i. e., the buoyant effect of the water, is always the same, no matter what the depth may be, and if the specific gravity of the body is heavier than the water, the body will sink until it meets an obstruction.

(359) Applying rule **110**, Art. **595**,

$$\text{new volume} = \frac{1.6 \times 90}{21} = 6\frac{6}{7} \text{ cu. ft.} \quad \text{Ans.}$$

(360) Apply rule **112**, Art. **626**.

$$\text{Area} = \frac{12,000}{5,000} = 2.4 \text{ sq. in.}$$

$$\text{Diameter} = \sqrt{\frac{2.4}{.7854}} = 1.74'' +. \quad \text{Ans.}$$

(361) Apply rule **120**, Art. **633**.

$$\text{Load} = 1,000 \times 4.75^2 = 22,562.5 \text{ lb.} \quad \text{Ans.}$$

(362) First calculate the load it will sustain in the middle by means of rule **124**, Art. **644**.

$$\text{Load in middle} = \frac{4 \times 10^2 \times 8 \times 30}{28} = 3,428\frac{4}{7} \text{ lb.}$$

$$\text{Uniform load} = 3,428\frac{4}{7} \times 2 = 6,857\frac{1}{7} \text{ lb.} \quad \text{Ans.}$$

(363) Apply rule **127**, Art. **653**. (See Table 25.)

$$\text{Horsepower} = \frac{10^3 \times 200}{70} = 2,857\frac{1}{7}. \quad \text{Ans.}$$

(364) Area of cross-section of pipe $= \left(\frac{3}{8}\right)^2 \times .7854 =$ $.1104$ sq. in. Weight of water in pipe $= .1104 \times (10 \times 12) \times .03617 = .47918$ lb. Total weight causing pressure on the upper cylinder head $= 25 + \frac{10}{16} + .47918 = 26.10418$ lb.

(*a*) Pressure per sq. in. on the top $= 26.10418 \div .1104 =$ 236.45 lb. Ans.

(*b*) Pressure per sq. in. on the bottom $= 236.45 + 36 \times .03617 = 237.7521$ lb. Ans.

(*c*) Equivalent weight = total pressure on lower cylinder head $= 20^2 \times .7854 \times 237.75 = 74,691.54$ lb. Ans.

(365) $36 \div 2 = 18''$. $18 \times .03617 + 236.45 = 237.1$ lb. per sq. in., pressure per sq. in. against the plate. Total pressure $= 1^2 \times .7854 \times 237.1 = 186.22$ lb. Ans.

(366) Pressure in the condenser $= 30 - 20 = 10''$. Since a column of mercury $30''$ high produces a pressure of 14.7 lb. per sq. in., the pressure which a $10''$ column would produce is found by the proportion $30 : 10 :: 14.7 : x$. Hence, pressure in the condenser $= x = 4.9$ lb. per sq. in. Ans.

(367) Apply rule **109**, Art. **594**.

$$\text{New pressure} = \frac{52 \times (300 - 120)}{300} = 31.2 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(368) Apply rule **114**, Art. **628**.

$$\text{Load} = 18{,}000 \times .5^2 = 4{,}500 \text{ lb.} \quad \text{Ans.}$$

(369) Apply rule **119**, Art. **633**.

$$\text{Circumference} = .0408\sqrt{14{,}000} = 4.83'', \text{ nearly.} \quad \text{Ans.}$$

(370) Apply rule **125**, Art. **645**.

$$\text{Load} = \frac{4 \times 2^2 \times .6 \times 150}{6} = 480 \text{ lb.} \quad \text{Ans.}$$

(371) Apply rule **129**, Art. **655**. (See Table 25.

$$\text{Diameter} = \sqrt[3]{\frac{90 \times 1{,}000}{80}} = 10.4 \text{ in.} \quad \text{Ans.}$$

(372) (*a*) $12''$. Ans.

(*b*) $18''$. Ans.

(373) See Arts. **616-619**.

(374) Apply rule **114**, Art. **628**.

$$\text{Load} = 18{,}000 \times \left(\frac{13}{16}\right)^2 = 11{,}883 \text{ lb.} \quad \text{Ans.}$$

(375) Apply rule **120**, Art. **633**.

$$\text{Circumference} = .0316\sqrt{8{,}000} = 2.83''. \quad \text{Ans.}$$

(376) Apply rule **124**, Art. **644**, and multiply the result by 2.

$$\text{Load} = \frac{4 \times 6^2 \times 2 \times 160}{20} \times 2 = 4{,}608 \text{ lb.} \quad \text{Ans.}$$

(377) Apply rule **128**, Art. **654**.

$$R = \frac{65 \times 80}{4^2} = 81\frac{1}{4} \text{ revolutions per minute.} \quad \text{Ans.}$$

(378) See Art. **584**.

(379) See Arts. **620** to **622**.

(380) Apply rule **115**, Art. **628**.

$$\text{Load} = 12{,}000 \times \left(\frac{5}{8}\right)^2 = 4{,}687.5 \text{ lb} \quad \text{Ans.}$$

(381) Apply rule **118**, Art. **633**.

$$\text{Load} = 600 \times 6^2 = 21{,}600 \text{ lb.} \quad \text{Ans.}$$

(382) 4 ft. = 48″. Area to be sheared $= 48 \times \frac{1}{2} = 24$ sq. in. Applying rule **126**, Art. **649**,

$$\text{force} = 24 \times 40{,}000 = 960{,}000 \text{ lb.} \quad \text{Ans.}$$

(383) Applying rule **128**, Art. **654**,

$$\frac{70 \times 200}{7^3} = 40.8 \text{ revolutions per minute, nearly.} \quad \text{Ans}$$

STEAM AND STEAM BOILERS.

(QUESTIONS 384-444*g*.)

(384) See Arts. **656** to **660.**

(385) See Art. **657.**

(386) See Art. **667.**

(387) See Arts. **658** to **660.**

(388) See Arts. **661, 662, 666,** and **667.**

(389) See Arts. **658** to **660** and **667** to **669.**

(390) (*a*) and (*b*) See Arts. **663** and **664.**

(*c*) 1 B. T. U. = 778 ft.-lb.

$30\frac{1}{2}$ B. T. U. $= 30\frac{1}{2} \times 778 = 23{,}729$ ft.-lb. Ans.

(391) 35 H. P. $= 35 \times 33{,}000$ ft.-lb. per min. $= 35 \times 33{,}000 \times 60$ ft.-lb. per hour $= \dfrac{35 \times 33{,}000 \times 60}{778}$ B. T. U. per hour $= 89{,}074.5$ B. T. U.

But this is the heat actually used, or 20% of the whole. Hence, the heat required is $89{,}074.5 \div .20 = 445{,}372.5$ B. T. U. Ans.

(392) One horsepower $= 33{,}000 \times 60$ ft.-lb. per hour.

$$= \frac{33{,}000 \times 60}{778} \text{ B. T. U. per hour.}$$

Each pound of coal gives 14,000 B. T. U., of which 8%, or $14{,}000 \times .08 = 1{,}120$ B. T. U., are utilized. Hence, the coal required per hour per H. P. is

$$\frac{33{,}000 \times 60}{778} \div 1{,}120 = 2.27 \text{ lb. Ans.}$$

(393) By rule **130**, B. T. U. $= c\,W(t' - t) = .2026 \times 22\frac{1}{2} \times (68 - 44) = 109.4$ B. T. U. Ans.

(394) (*a*) See Art. **668**.

(*b*) To raise the ice from 17° to 32° requires for each pound $.504 \times (32 - 17) = 7.56$ B. T. U. To melt it requires 144 B. T. U. Hence, 1 lb. requires $144 + 7.56 = 151.56$ B. T. U.; and 11 lb. requires $11 \times 151.56 = 1,667.16$ B. T. U. Ans.

(395) By rule **130**, B. T. U. $= c\,W(t' - t) = .4805 \times 6 \times (342 - 310) = 92.256$ B. T. U. Ans.

(396) Use rule **131**. The product of the weight, specific heat, and temperature of the copper is $18 \times .0951 \times 305 = 522.099$; of the iron, $13 \times .1138 \times 278 = 411.2732$; of the water, $32 \times 1 \times 56 = 1,792$. The sum is 2,725.3722. The product of the weight and specific heat of the copper is $18 \times .0951 = 1.7118$; of the iron, $13 \times .1138 = 1.4794$; of the water, $32 \times 1 = 32$. The sum is 35.1912. Hence, the resulting temperature is $\frac{2,725.3722}{35.1912} = 77.45°$. Ans.

(397) (*a*) 966.069 B. T. U. Ans.

(*b*) The total heat of steam above 32° at 212° is 1,146.6 B.T.U. $1,146.6 - (63 - 32) = 1,115.6$ B.T.U. = heat necessary to change 1 lb. of water at 63° into steam at 212°. Hence, for 8 lb. of water $1,115.6 \times 8 = 8,924.8$ B. T. U. are required. Ans.

(398) To raise the temperature of 1 lb. of ice from 23° to 32° requires $(32 - 23) \times .504 = 4.536$ B. T. U. To melt the ice requires 144 B. T. U. To change the water at 32° to steam at 212° requires 1,146.6 B. T. U. per pound (see table). $1,146.6 + 144 + 4.536 = 1,295.136$ B. T. U. per pound. For 2.2 lb., $1,295.136 \times 2.2 = 2,849.3$ B. T. U. are required. Ans.

(399) See Arts. **674** and **675**.

(400) See Arts. **676** and **678**.

(401) The volume of a pound of steam at 80 lb. pressure, absolute, is 5.358 cu. ft. At 82 lb. the volume is 5.235 cu. ft. (see steam table). Therefore, at 81 lb. pressure the volume is $\frac{5.358 + 5.235}{2} = 5.2965$ cu. ft. $5.2965 \times 6 = 31.779$ cu. ft. Ans.

(402) From the steam table, the absolute pressure is 60 lb. per sq. in., nearly. Hence, the gauge pressure is $60 - 14.7 = 45.3$ lb. per sq. in. Ans.

(403) $135 - 50 = 85$ = heat units necessary to raise the temperature of 1 lb. of water from 50° to 135°. Total heat of steam at 78 lb absolute pressure = 1,176.529 B. T. U. $1{,}176.529 - (135 - 32) = 1{,}073.529$ B. T. U. given up by the steam in condensing and falling to a temperature of 135°. Hence, $85 \times 250 \div 1{,}073.529 = 19.795$ lb. Ans.

(404) Referring to the steam table, 1 lb. of steam at 350° occupies a volume of 3.272 cu. ft. $6\frac{1}{8} \times 3.272 = 20.041$ cu. ft. = volume of $6\frac{1}{8}$ lb. of steam. Ans.

(405) (*a*) See Art. **679.**
(*b*) See Art. **679.**

(406) See Arts. **684** to **686.**

(407) See Art. **688.**

(408) See Arts. **689** and **690.**

(409) See Arts. **689** and **690.**

(410) See Arts. **692** and **693.**

(411) (*a*) See Art. **691.**
(*b*) See Art. **698.**

(412) See Art. **700.**

(413) First find the percentage of reduction. Thus, $.625 : .375 :: 100 : x = 60\%$, the ratio the area at the point of fracture bears to the original area. Hence, the reduction in area $= 100 - 60 = 40\%$.

The legal ductility (see Art. **702**) is

$$15 + \left(\frac{53,000 - 45,000}{1,000}\right) = 23\%;$$

hence, the plate may be used.

(414) See Art. **717.**

(415) (*a*) By rule **133**, $L = A\ P$. Hence, the load $= 7.25 \times 7.25 \times 150 = 7,884.375$ lb. Ans.

(*b*) Taking the legal stress per sq. in. of section at 6,000 lb., the area of the stay will be $7,884.375 \div 6,000 = 1.314$ sq. in. The corresponding diameter is $1\frac{5}{16}$ in., nearly. Ans.

(416) By rule **138**, $a = \sqrt{\frac{120\ t^2}{P}}$.

The gauge pressure $= 90 - 14.7 = 75.3$ lb.
Substituting, we have

$$a = \sqrt{\frac{120 \times 8^2}{75.3}} = 10 \text{ in., nearly.} \quad \text{Ans.}$$

(417) By rule **137**, $P = \frac{120\ t^2}{a^2}$.

Substituting, we have $P = \frac{120 \times 8^2}{8.5^2} = 106.3$ lb. Ans.

(418) By rule **135**, $P = \frac{B\,M}{A}$.
Substituting, we get

$$P = \frac{.875^2 \times .7854 \times 6,000}{7.25^2} = 68.64 \text{ lb.} \quad \text{Ans.}$$

(419) (*a*) By rule **133**, $L = A\ P$. Hence, the load $= 14.5 \times 15 \times 170 = 36,975$ lb. The stress therefore $=$

$$\frac{36,975}{2.375^2 \times .7854} = 8,346.3 \text{ lb.} \quad \text{Ans.}$$

(*b*) The stays should not be used. For the reason, see Art. **727.**

(420) Draw to any convenient scale a right-angled triangle, as $A\,B\,C$, Fig. 16, making the sides adjacent to the right angle 4 ft. 8 in. and 12 in., respectively. Next,

draw a line EF of sufficient length parallel to BC. On this line lay off the pressure on the head to any convenient scale, say $\frac{1}{32}$ in. = 200 lb. Call the two extremities of this line a and b. Draw a line parallel to AC through the point

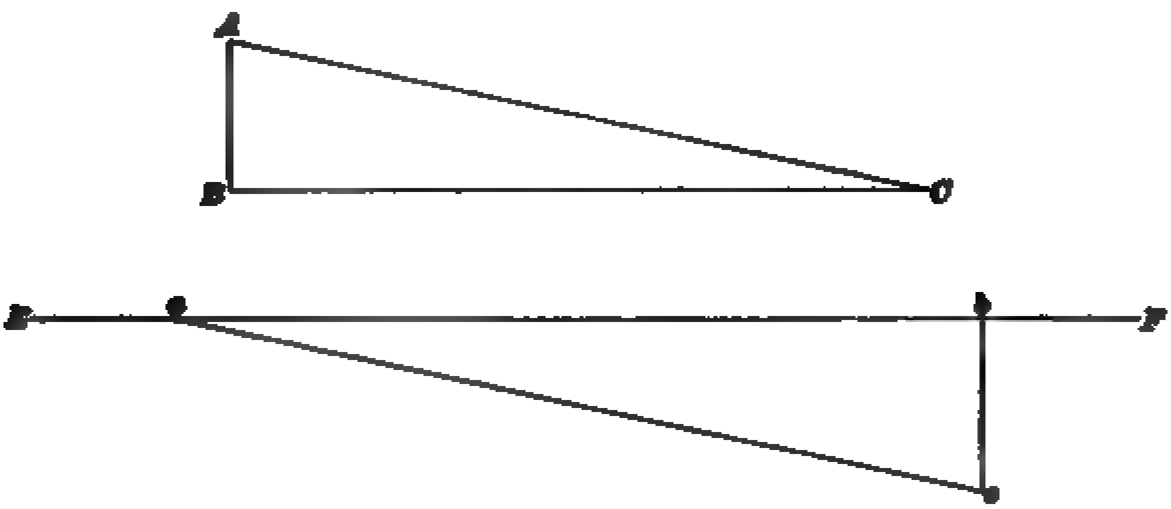

FIG. 16.

a, and a line parallel to AB through the point b. Call the point where these two lines intersect, c. Then, the length of the line ac, measured to the same scale as the line ab, represents the stress. Measuring it, it is found to be $\frac{88}{32}$ in. in length. Hence, the stress $= 88 \times 200 = 17{,}600$ lb. Ans.

(421) See Arts. **683** and **737**.

(422) By rule **142**,

$$P = \frac{89{,}600\, T^2}{L D}.$$

Substituting, we have

$$P = \frac{89{,}600 \times .5^2}{3.25 \times 39} = 176.73 \text{ lb.} \quad \text{Ans.}$$

(423) By rule **144**,

$$P = \frac{14{,}000\, T}{D}.$$

Substituting, we have

$$P = \frac{14{,}000 \times .4375}{38.5} = 159.09 \text{ lb.} \quad \text{Ans.}$$

(424) By rule **145**,

$$P = \frac{T S}{2.5 R}.$$

Substituting, we have

$$P = \frac{.625 \times 52,000}{2.5 \times 48} = 270.83 \text{ lb.} \quad \text{Ans.}$$

(425) By rule **149**,

$$P = \frac{T S}{6 R}.$$

Substituting, we have

$$P = \frac{.375 \times 65,000}{6 \times 48} = 84.63 \text{ lb.} \quad \text{Ans.}$$

(426) By rule **148**,

$$P_b = \frac{T S}{R}.$$

Substituting, we have

$$P_b = \frac{.5 \times 17,800}{5} = 1,780 \text{ lb.} \quad \text{Ans.}$$

(427) (*a*) See Art. **756.**

(*b*) See Arts. **757** to **760.**

(428) See Art. **760.**

(429) The area of the valve $= 2.5^2 \times .7854 = 4.9087$ sq in. Hence, the upward force $= 4.9087 \times 60 = 294.52$ lb.

The weight (see Art. **765**) must be equal to the upward force. Hence, the weight $= 294.52$ lb. Ans.

(430) First find the weight of the lever. Its cubic contents are $42 \times 1.5 \times .75 = 47.25$ cu. in. Its weight, therefore, is $47.25 \times .28 = 13.23$ lb. The distance from the fulcrum to center of gravity of the lever is one-half its length, since the lever is straight; hence, the downward force due to the weight of the lever (from Art. **766**)

$= \frac{21 \times 13.23}{4} = 69.46$ lb. The total downward force due to the weight of the valve, stem, and lever is, therefore, $69.46 + 5.54 = 75$ lb.

By rule **153**,

$$W = \frac{(A\,P - w)\,D}{L}.$$

Substituting, we have

$$(a) \quad \frac{(10 \times 65 - 75) \times 4}{40} = 57.5 \text{ lb.} \quad \text{Ans.}$$

$$(b) \quad \frac{(10 \times 75 - 75) \times 4}{40} = 67.5 \text{ lb.} \quad \text{Ans.}$$

$$(c) \quad \frac{(10 \times 85 - 75) \times 4}{40} = 77.5 \text{ lb.} \quad \text{Ans.}$$

$$(d) \quad \frac{(10 \times 100 - 75) \times 4}{40} = 92.5 \text{ lb.} \quad \text{Ans.}$$

(431) (*a*) See Art. **769.**

(*b*) See Art. **771.**

(*c*) See Art. **771.**

(432) (*a*) See Art. **773.**

(*b*) See Arts. **774** and **775.**

(*c*) See Art. **774.**

(433) See Art. **776.**

(434) (*a*) See Art. **777.**

(*b*) See Art. **778.**

(*c*) See Art. **777.**

(435) See Art. **780.**

(436) See Art. **781.**

(437) (*a*) See Arts. **784** and **785.**

(*b*) See Art. **785.**

(438) See Art. **789.**

(439) See Arts. **791** to **793.**

(440) See Art. **794.**

(441) See Arts. **795** to **797.**

(442) See Art. **786.**

(443) See Art. **788.**

(444) See Art. **789.**

(444*a*) (*a*) By rule **146,**

$$P = \frac{T\,S}{5\,R}.$$

Substituting, we have

$$P = \frac{\frac{9}{16} \times 45{,}000}{5 \times 42} = 120.5 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(*b*) By rule **149,**

$$P = \frac{T\,S}{6\,R}.$$

Substituting, we get

$$P = \frac{\frac{5}{16} \times 56{,}000}{6 \times 18} = 162 \text{ lb. per sq. in.}$$

But as the working pressure allowed is that allowed on the weakest part, which, according to the calculations, is the head, the pressure that may be carried is 120.5 lb. per sq. in. Ans.

(444*b*) By rule **135*b*,**

$$L = \frac{l}{b} A\,P.$$

Substituting, we get

$$L = \frac{28}{25} \times 160 \times 196 = 35{,}184 \text{ lb.} \quad \text{Ans.}$$

(444*c*) By rule **132*i*,**

$$V = \frac{\sqrt{(11\,p + 4\,d)\,(p + 4\,d)}}{10}.$$

Substituting, we have

$$V = \frac{\sqrt{(11 \times 3 + 4 \times \frac{7}{8}) \times (3 + 4 \times \frac{7}{8})}}{10} = 1.54 \text{ in.} \quad \text{Ans.}$$

(444*d*) By rule **132*j***,

$$p_d = \frac{6\,p + 4\,d}{10}.$$

Substituting, we get

$$p_d = \frac{6 \times 3 + 4 \times \frac{7}{8}}{10} = 2.15 \text{ in.} \quad \text{Ans.}$$

(444*e*) By rule **132*l***,

$$p_m = 2.62\,T + 1\tfrac{5}{8}''.$$

Substituting values, we get

$p_m = 2.62 \times \frac{5}{8} + 1\frac{5}{8}'' = 3.263$ in., the maximum pitch of rivets allowable with a plate of the given thickness and a double-riveted lap-joint. Hence, the joint would not be allowed to be used. Ans.

(444*f*) As the rule for finding the distance between the rows of rivets calls for the diameter of the rivet, this must be found first by rule **132*b***,

$$d = T + \tfrac{5}{16}''.$$

Substituting values, we have

$$d = \tfrac{5}{8} + \tfrac{5}{16} = \tfrac{15}{16} \text{ of an inch.}$$

Then, by rule **132*h***,

$$V = \frac{4\,d + 1}{2}.$$

Substituting, we get

$$V = \frac{4 \times \frac{15}{16} + 1}{2} = 2.375 \text{ in.} \quad \text{Ans.}$$

(444*g*) (*a*) By rule **132*f***,

$$p = \frac{23 \times .7854\,d^2\,n}{28\,T} + d.$$

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Substituting, we have

$$p = \frac{23 \times .7854 \times (\frac{13}{16})^2 \times 2}{28 \times \frac{7}{16}} + \frac{13}{16} = 2.759 \text{ in.} \quad \text{Ans.}$$

(*b*) By rule **132*l***,

$$p_m = 2.62\,T + 1\tfrac{5}{8}''.$$

Substituting, we get

$$p_m = 2.62 \times \tfrac{7}{16} + 1\tfrac{5}{8}'' = 2.771 \text{ in.} \quad \text{Ans.}$$

STEAM AND STEAM BOILERS.

(QUESTIONS 445–499.)

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(445) See Arts. **798** to **806.**

(446) See Art. **802.**

(447) See Art. **804.**

(448) See Art. **805.**

(449) See Art. **806.**

(450) See Art. **807.**

(451) See Art. **808.**

(452) See Art. **809.**

(453) See Art. **811.**

(454) See Arts. **813** and **814.**

(455) See Art. **815.**

(456) See Art. **817.**

(457) First correct the boiling point. $30 - 29.8 = .2$. $2 \times .16 = .32$. From Art. **817,** we find that the water would boil at 30 in. at $219.4 + .32 = 219.72°$ F. Then, by rule **154,**

$$S = \frac{(A \pm B) - 212}{1.2}.$$

Substituting, we have

$$S = \frac{219.72 - 212}{1.2} = \frac{6.43}{32} \text{ saturation.} \quad \text{Ans.}$$

(458) See Art. **818.**

(459) (*a*) See Art. **818.**

(*b*) See Art. **818.**

(460) By rule **155**, $A = \frac{C \times D}{B}$.

Substituting, we get

$$A = \frac{8,500 \times 1}{2\frac{1}{2}} = 3,400 \text{ lb.}$$

As there are 8.4 pounds in one gallon, $\frac{3,400}{8.4} = 404.76$ gallons. Ans.

(461) By rule **156**, $E = A\left(\frac{B}{D} - 1\right)$.

Substituting, we have $E = 1\left(\frac{5\frac{1}{8}}{1} - 1\right) = 4\frac{1}{8}$ lb. Ans.

(462) See Art. **844.**

(463) See Art. **846.**

(464) (*a*) See Arts. **849** and **850.**

(*b*) See Arts. **847** and **848.**

(465) (*a*) See Art. **852.**

(*b*) See Arts. **852, 857, 859,** and **860.**

(466) By rule **164**, $W = 3\sqrt{H}$.

Substituting, we have

$W = 3\sqrt{81} = 27$ lb. per sq. ft. of grate surface per hour.

The total grate surface $= 6 \times 4 \times 3\frac{3}{4} \times 6 = 540$ sq. ft. Hence, the coal consumption per hour $= 540 \times 27 = 14,580$ lb. The time of the trip reduced to hours $= \frac{2,282}{14} = 163$ hours. Consequently, the coal consumption for the trip $= 14,580 \times 163 = 2,376,540$ lb., or 1,188 tons 540 lb. Ans.

(467) See Arts. **857** and **858.**

(468) (*a*) (*b*) (*c*) See Arts. **863** to **865.**

(469) See Art. **870.**

(470) (*a*) (*b*) (*c*) See Arts. **872, 873,** and **875.**

(471) Arrange the numbers of the furnaces as stated in Art. **876.** See Fig. 17.

Fire	1	2	3	4	5	6
Slice	2	3	4	5	6	1
Level	3	4	5	6	1	2
Clean Grate.	4	5	6	1	2	3

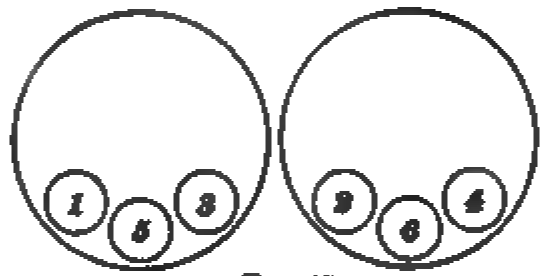


FIG. 17.

(472) (*a*) (*b*) See Arts. **833** and **834.**

(473) See Arts. **824** to **826,** and **832.**

(474) (*a*) (*b*) See Art. **839.**

(475) According to rule **158,** the air required for 1 pound of fuel is

$$A = 1.52\,(C + 3\,H)$$
$$= 1.52\,(94 + 3 \times .5) = 145.16 \text{ cu. ft.}$$

Then, 17 lb. would require $17 \times 145.16 = 2{,}467.7$ cu. ft. Ans.

(476) Rule **159** gives

$$B = 145\,C + 620\,H$$
$$= 145 \times 94 + 620 \times .5 = 13{,}940 \text{ B. T. U.}$$

Ans.

(477) Using rule **160,** we have

$13{,}675 \div 966 = 14.16$ lb. of water evaporated from and at 212°. Ans.

(478) (*a*) HNO_3.

$$\left.\begin{array}{l} 1 \times 1 = 1 \text{ part hydrogen} \\ 1 \times 14 = 14 \text{ parts nitrogen} \\ 3 \times 16 = \underline{48} \text{ parts oxygen} \\ \quad\quad\quad\;\; 63 \end{array}\right\} \text{by weight.}$$

The substance is then composed by weight of

$$\frac{1}{63} = 1.59\% \text{ hydrogen,}$$

$$\frac{14}{63} = 22.22\% \text{ nitrogen,}$$

$$\frac{48}{63} = 76.19\% \text{ oxygen.}$$

$$\overline{100.00}$$

(*b*) N_2O_3. $2 \times 14 = 28$ $\frac{28}{76} =$ 36.84% nitrogen

$3 \times 16 = 48$ $\frac{48}{76} =$ 63.16% oxygen.

$\overline{76}$ $\overline{100.00}$

(*c*) H_2SO_4. $2 \times 1 = 2$ $\frac{2}{82} =$ 2.44% hydrogen,

$1 \times 32 = 32$ $\frac{32}{82} =$ 39.02% sulphur,

$3 \times 16 = 48$ $\frac{48}{82} =$ 58.54% oxygen.

$\overline{82}$ $\overline{100.00}$

(479) (*a*) See Art. **834.**

(*b*) See Art. **837.**

(*c*) One pound of carbon requires 5.8 lb. of air to burn it to C O gas. Eight pounds of carbon, therefore, require $8 \times 5.8 = 46.4$ lb. of air. Ans.

(480) See Arts. **841** and **842.**

(481) Since the product is SO_2, the composition of this product by weight is

$$S = 1 \times 32 = 32$$
$$O = 2 \times 16 = 32$$

or 50% of each. Hence, 1 pound of sulphur requires a pound of oxygen.

The scheme is, therefore, as follows:

1 lb. sulphur	sulphur	1 lb.	2	lb. SO_2
4.35 lb. air	oxygen	1 lb.		
	nitrogen	3.35 lb.	3.35	lb. nitrogen
		5.35	5.35	

(482) See Arts. **833** and **829.**

(483) (*a*) See Art. **879.**

(*b*) See Art. **879.**

(484) See Art. **880.**

(485) (*a*) See Art. **881.**
(*b*) See Art. **881.**

(486) (*a*) See Art. **880.**
(*b*) See Art. **882.**

(487) See Arts. **885** to **888.**

(488) (*a*) See Art. **890.**
(*b*) See Art. **891.**

(489) See Art. **892.**

(490) See Art. **893.**

(491) See Art. **894.**

(492) The coal burned per square foot of grate surface per hour is found by rule **161.**

$$W = 2\sqrt{H} - 1 = 2\sqrt{64} - 1 = 15 \text{ lb.}$$

The evaporation per pound of fuel, as given in Table 30, is 8.75 pounds.

(*a*) Then, by rule **165,** $G = \frac{W}{F \times E}$.
Substituting, we have

$$G = \frac{6,500}{15 \times 8.75} = 49.52 \text{ sq. ft.} \quad \text{Ans.}$$

(*b*) By rule **166,** the heating surface equals

$$49.52 \times 42 = 2,079.84 \text{ sq. ft.} \quad \text{Ans.}$$

(493) See Arts. **903** to **913.**

(494) See Art. **899.**

(495) See Art. **915.**

(496) See Art. **918.**

(497) (*a*) See Art. **929.**
(*b*) See Art. **928.**

(498) By rule **168,**

$$Q = \frac{x}{w} = \frac{1}{l}\left[\frac{W}{w}(t_1 - t_2) - (t - t_1)\right].$$

First find the latent heat of steam at the observed pressure, viz., 90 pounds, gauge. From the steam table this is found to be 881.368, say 881.4 B. T. U. Next find the temperature of the steam at the observed pressure. This, from the steam tables, is 330.956, say 331° F.

Substituting values, we have

$$Q = \frac{1}{881.4}\left[\frac{400}{35.2}(135 - 44) - (331 - 135)\right] = .9508 = 95.08\%. \text{ Ans.}$$

(499) (*a*) First find the water evaporated into dry steam.

$$114,962.7 \times .98 = 112,663.45 \text{ lb.}$$

Then, the evaporation per pound of coal equals

$$112,663.45 \div 12,000 = 9.389 \text{ lb. Ans.}$$

(*b*) The water evaporated per pound of combustible equals

$$112,663.45 \div \left[12,000\left(\frac{100-4.81}{100}\right)\right] = 112,663.45 \div 11,422.8 = 9.863 \text{ lb. Ans.}$$

(*c*) By rule **167**, $W_1 = \frac{W(H - t + 32)}{966.1}$.

First find the total heat of one pound of steam at the gauge pressure. This, from the steam tables, is found to be 1,176.715 B. T. U.

Substituting, we have

$$W_1 = \frac{112,663.45 \times (1,176.715 - 132 + 32)}{966.1} = 125,563.01 \text{ lb.}$$

evaporated from and at 212° F.

Hence, the equivalent evaporation per pound of coal equals

$$125,563.01 \div 12,000 = 10.463 \text{ lb. Ans.}$$

(*d*) The equivalent evaporation per pound of combustible equals

$$125,563.01 \div 11,422.8 = 10.992 \text{ lb. Ans.}$$

STEAM ENGINES.

(PART 1.)

(500) (*a*) See Arts. **936** and **937.**

(*b*) It is not necessary to know either the diameter or length of cylinder, as they are both included in the volume.

(501) (*a*) From rule **169,**

$$W = 144\,p\,V = 144 \times 21 \times 24 = 72{,}576 \text{ ft.-lb. Ans.}$$

(*b*) Work per minute $= 72{,}576 \times 60$ ft.-lb. Hence, horsepower $= \dfrac{72{,}576 \times 60}{33{,}000} = 131.96$ H. P. Ans.

(502) (*a*) $p = .57 \times 60 = 34.2$ lb. per sq. in. $V = 2.9 \times 1.3 = 3.77$ cu. ft. From rule **169,** work done per stroke $= W = 144\,p\,V = 144 \times 34.2 \times 3.77 = 18{,}566.5$ ft.-lb. Ans.

(*b*) Work per minute $= 18{,}566.5 \times 74$ ft.-lb. Hence, horsepower $= \dfrac{18{,}566.5 \times 74}{33{,}000} = 41.63$ H. P. Ans.

(503) See Arts. **944** and **945.**

(504) See Arts. **946** to **948.**

(505) See Arts. **948** and **949.**

(506) See Art. **947.**

(507) See Arts. **956** to **958.**

(508) See Art. **955.**

(509) See Art. **958.**

(510) See Arts. **958** and **960.**

(511) See Arts. **963, 971,** and **973**

(512) See Arts. **971** and **972.**

(513) See Arts. **967** to **969.**

(514) (*a*) The construction is shown in Fig. 18. The distance $EF = 3\frac{1}{2}'' =$ travel of valve. Upon EF as a diam-

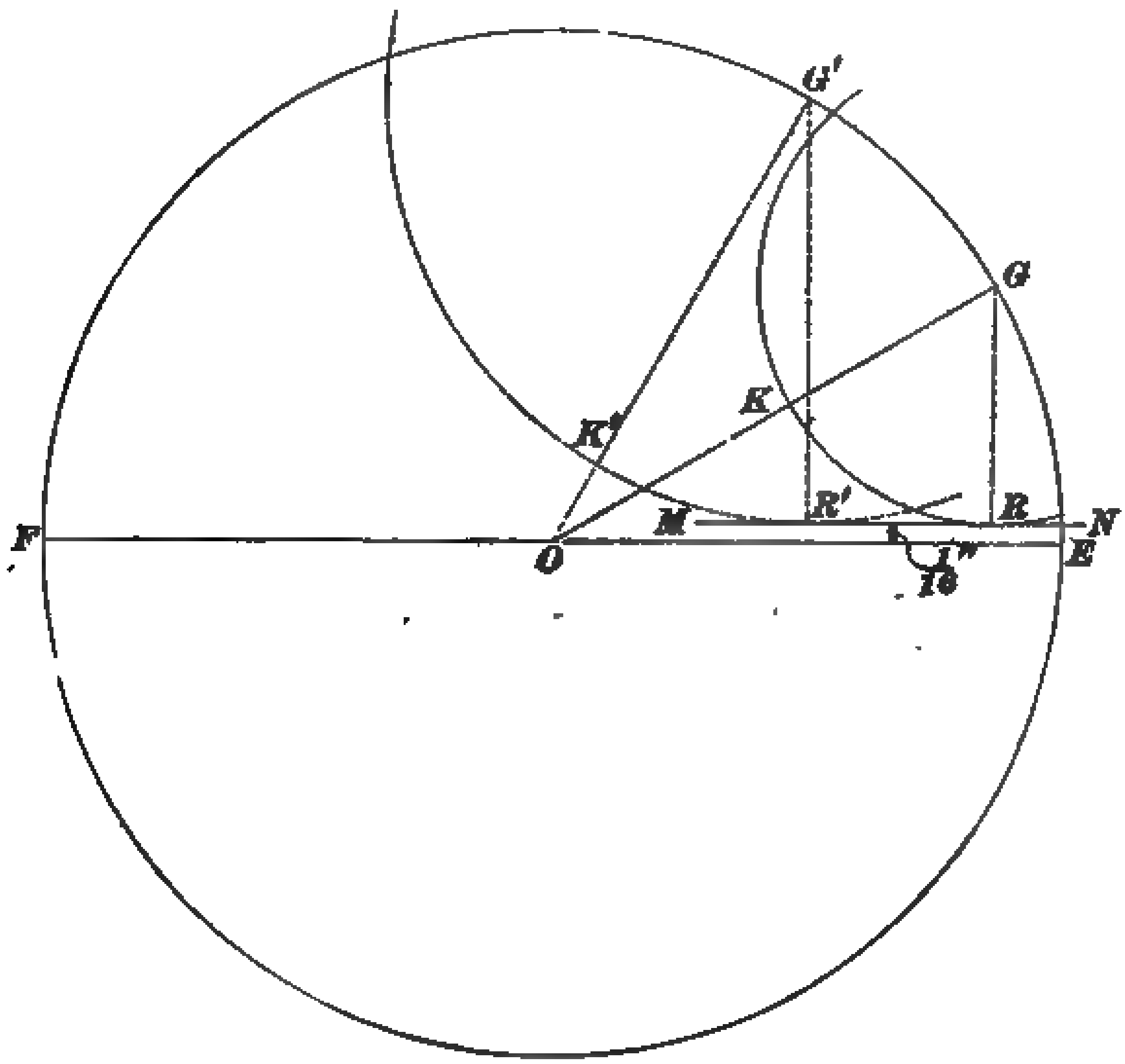


FIG. 18.

eter, describe the circle. Draw MN parallel to EF and $\frac{1}{16}''$ (= lead) above it. Lay off angle $EOG = 30° =$ angle of advance. Drop the perpendicular GR upon MN. Then, GR is the required lap. Upon being measured, it is found to be $\frac{13}{16}''$. Ans.

(*b*) With GR as a radius and a center G, describe a circle intersecting line OG in K. Then, $OK = \frac{15}{16}'' =$ port opening. Ans.

(515) (*a*) In case the angle of advance is 60°, lay off EOG', Fig. 18, = 60°, and from G' drop a perpendicular $G'R'$ upon MN. Then, $G'R' = \text{lap} = 1\frac{7}{16} = \frac{23}{16}''$, nearly. The increase of lap is, therefore, $\frac{23}{16} - \frac{13}{16} = \frac{10}{16} = \frac{5}{8}''$. Ans.

(*b*) The port opening is, in this case, $OK' = \frac{5}{16}''$, nearly. Ans

(516) See Arts. **976** and **981.**

(517) 20 : 28 :: eccentric throw : valve travel;
or, 20 : 28 :: 4 : valve travel.

Hence, valve travel $= \frac{4 \times 28}{20} = 5.6$ in. Ans.

(518) See Arts. **978** to **980.**

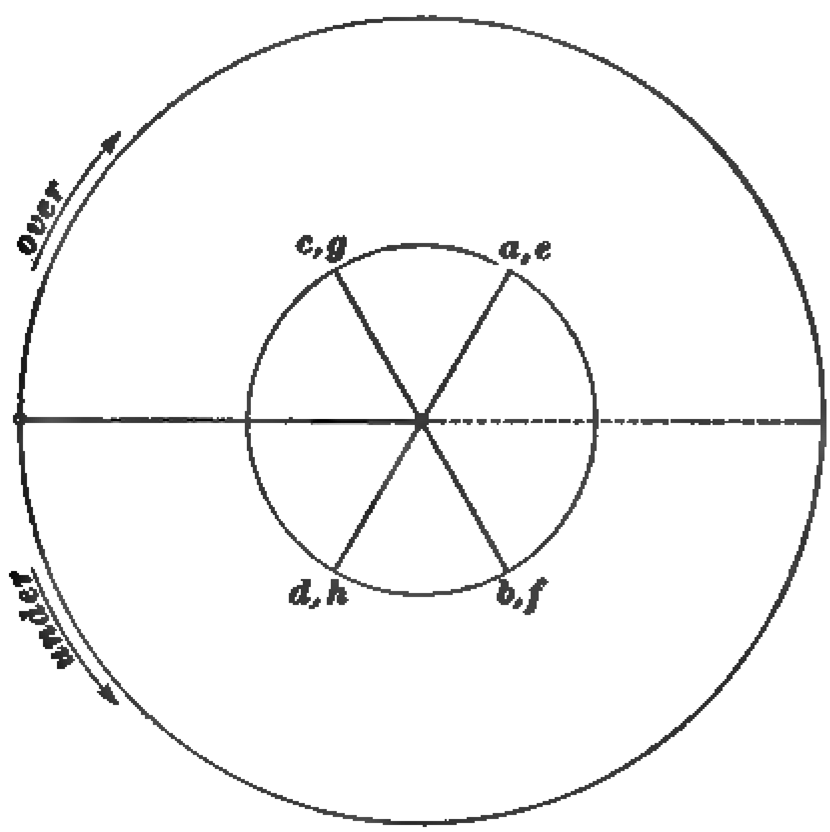


FIG. 19.

(519) The cases from *a* to *h* are shown in Fig. 19.

(520) See Arts. **982** and **983.**

(521) See Art. **984.**

(522) See Art. **1039.**

(523) (*a*) See Art. **1041.**

(*b*) See Art. **1042.**

(524) See Arts. **990** and **991.**

(525) (*a*) See Arts. **992** to **995.**

(*b*) According to rule **170,**

$$\text{Real cut-off} = \frac{s+i}{1+i} = \frac{\frac{3}{8}+.07}{1+.07} = \frac{.375+.07}{1.07} = \frac{.445}{1.07} = .416. \quad \text{Ans.}$$

$$\text{Ratio of expansion} = \frac{1}{\text{real cut-off}} = \frac{1}{.416} = 2.4. \quad \text{Ans.}$$

(526) (*a*) $\text{Apparent cut-off} = \frac{9}{40} = .225.$ Ans.

(*b*) $\text{Real cut-off} = \frac{s+i}{1+i} = \frac{.225+.03}{1.03} = \frac{.255}{1.03} = .2476.$ Ans.

(*c*) $\text{Number of expansions} = \frac{1.03}{.255} = 4.04.$ Ans.

(527) (*a*) The volume V of the cylinder is $.7854 \times 18^2 \times 40 = 10{,}178.8$ cu. in. Using rule **169,** $144\,p\,V = 144 \times 34.6 \times \frac{10{,}178.8}{1{,}728} = 29{,}348.87$ ft.-lb. per stroke. Ans.

(*b*) $\frac{29{,}348.87 \times 140}{33{,}000} = 124.51$ H. P. Ans.

(528) (*a*) The steam in the cylinder at cut-off is that included in 9 in. of the length of the cylinder and steam in clearance.

The clearance is 3% of the cylinder volume $= 10{,}178.8 \times .03 = 305.364$ cu. in.

The volume displaced by piston up to cut-off is $.7854 \times 18^2 \times 9 = 2{,}290.23$ cu. in.

Volume of steam admitted per stroke $= 305.364 + 2{,}290.23 = 2{,}595.594$ cu. in.

At 100 lb., absolute, 1 cu. ft. of steam weighs .230293 lb. Hence, the steam per stroke is $\frac{2{,}595.594}{1{,}728} \times .230293 = .3459$ lb. Ans.

(*b*) To find the steam per H. P. per hour, multiply (*a*) by number of strokes per hour and divide by H. P., or

$.3459 \times 140 \times 60 \div 124.51 = 23\frac{1}{3}$ lb. steam per H. P. per hour. Ans.

(529) See Arts. **941** and **942.**

(530) See Arts. **977** and **980.**

(531) In cases (*a*) and (*c*) the eccentric is behind the crank $90°$ — angle of advance. Hence, the angle between eccentric and crank is $90° - 37° = 53°$. Ans.

In cases (*b*) and (*d*), the eccentric is ahead of the crank $90° +$ angle of advance $= 90° + 37° = 127°$. Ans.

(532) See Arts. **997** and **998.**

(533) See Arts. **999, 1000, 1003, 1006** to **1008.**

(534) The speed of the vessel in feet per minute is $\frac{6{,}082 \times 15}{60} = 1{,}520.5$ feet. Since the effective diameter of the wheels is 14 feet, and since they make 40 revolutions per minute, the velocity at which a stream is projected by them is $14 \times 3.1416 \times 40 = 1{,}759.296$ feet per minute. Applying rule **171**, we have

$$100 \times \frac{1{,}759.296 - 1{,}520.5}{1{,}759.296} = 13.57\%, \text{ nearly. Ans.}$$

(535) Applying rule **173**, we have

$$\frac{6{,}082 \times 15}{3.1416 \times 40 \times 60} = 12.1 \text{ feet, nearly. Ans.}$$

(536) See Art. **1007.**

(537) See Arts. **1011, 1012,** and **1009.**

(538) See Art. **1013.**

(539) See Arts. **1015** to **1017.**

(540) See Arts. **1018, 1019,** and **1021.**

(541) Applying rule **175**, we have

$$\frac{33{,}000 \times 1{,}020}{20 \times 70} = 24{,}042.86 \text{ lb. Ans.}$$

(542) First find the cross-sectional area of the stream projected by the screw propeller. This is $18^2 \times .7854 - 4^2 \times .7854 = 241.91$ square feet. Next, the velocity of the stream, in regard to the vessel, is $\frac{20 \times 70}{60} = 23.33$ feet per second. The velocity of the vessel in feet per second is $\frac{13 \times 6,082}{60 \times 60} =$ 21.96 feet, nearly. Hence, the velocity of the stream, in regard to the surrounding water, is $23.33 - 21.96 = 1.37$ feet per second. Then, the weight of the stream per second is $241.91 \times 23.33 \times 64.1 = 361,765.04$ lb. Now, applying rule **174**, we have

$$\frac{361,765.04 \times 1.37}{32.16} = 15,411 \text{ lb. Ans.}$$

(543) Applying rule **176**, we have

$$\frac{33,000 \times 430}{3.1416 \times 18 \times 40} = 6,273.3 \text{ lb. Ans.}$$

(544) From the table of constants, the constant $B =$ 240. Substituting values in rule **177**, we have

$$\frac{\sqrt[3]{8,916.6^2 \times 17^3}}{240} = 8,802 \text{ I. H. P., nearly. Ans.}$$

(545) Applying rule **178**, we have

$$\frac{80 \times 18}{20} = 72 \text{ revolutions per minute. Ans.}$$

(546) Applying rule **179**, we have

$$\frac{85^3 \times 8,000}{80^3} = 9,595.7 \text{ I. H. P. Ans.}$$

(547) See Art. **1035.**

(548) See Art. **1034.**

(549) See Art. **1038.**

(550) See Art. **1037.**

(551) See Art. **1040.**

STEAM ENGINES.

(PART 2.)

(552) (*a*) See Art. **1069.** (*b*) By rule **182,**

$$\text{piston speed} = \frac{l R}{6} = \frac{12 \times 120}{6} = 240 \text{ ft.} \quad \text{Ans.}$$

(553) See Art. **1073.**

(554) See Arts. **1067, 1071,** and **1072.**

(555) Admission, cut-off, release, and compression are all too late. The back pressure is also excessive. Add lap to the valve, shift the eccentric ahead on the shaft, and make the exhaust port or exhaust pipe larger.

(556) See Art. **1058.**

(557) See Arts. **1059** and **1060.**

(558) See Arts. **1074** to **1078.**

(559) See Arts. **1080, 1081,** and **1083.**

(560) Absolute pressure of entering steam is $65 + 14.7 = 79.7$ lb. per sq. in.; of exhaust, $14.7 + 2.3 = 17$ lb.

Temperature of entering steam (from table) is 311.6°, nearly.

Temperature of exhaust steam is 219.45°.

Absolute temperature T_1 of entering steam is $311.6 + 460 = 771.6°$.

Absolute temperature T_2 of exhaust is $219.45 + 460 = 679.45°$.

Thermal efficiency $= \frac{T_1 - T_2}{T_1} = \frac{771.6 - 679.45}{771.6} = \frac{92.15}{771.6} =$ 11.94%. Ans.

(561) See Arts. **1085** to **1088,** and **1084.**

(562) See Art. **1085.**

(563) The M. E. P. is found as shown in Fig. 20.

The middle ordinates are drawn as explained in Art. **1064.**

These ordinates are measured and multiplied by the scale of the spring, which reduces them from inches to pounds per square inch. In the figure the sum of the 20 ordinates

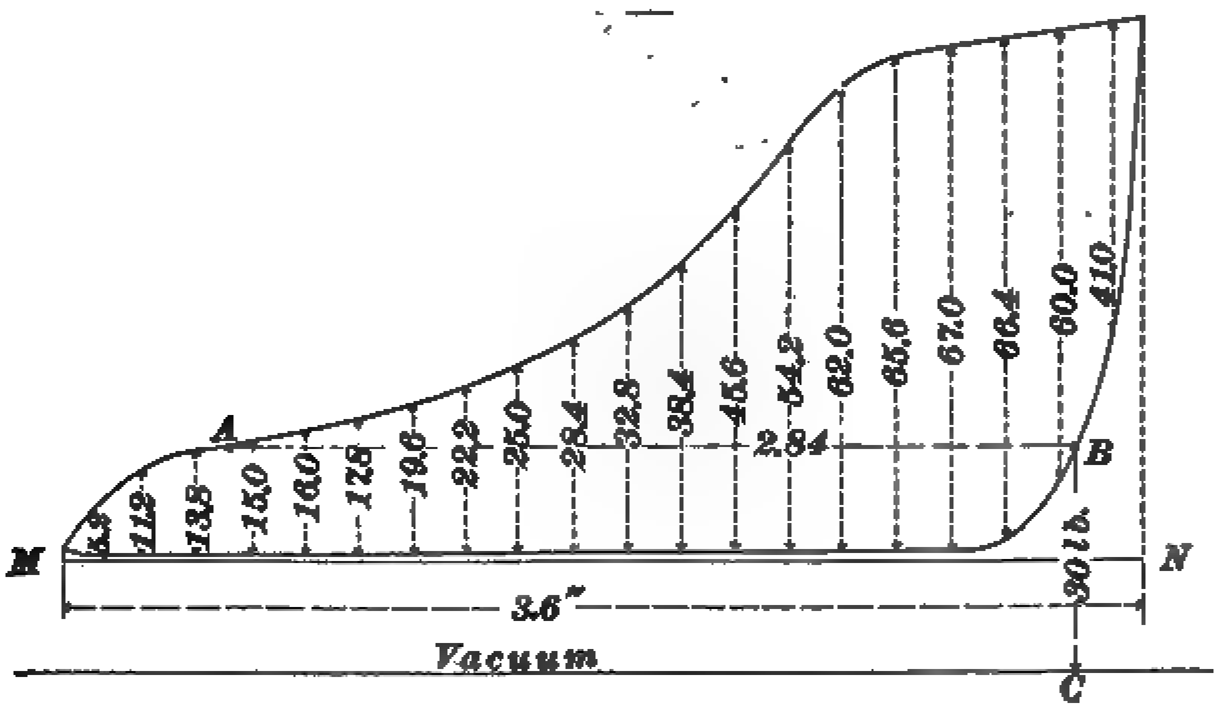


FIG. 20.

is 707.2 lb. The mean ordinate, which is the same thing as the M. E. P., is, therefore, $\frac{707.2}{20} = 35.36$ lb. Draw the vacuum line at a distance of $\frac{14.7}{40} = .3675''$ below the atmospheric line MN. Choose the point A near the point of release, and draw AB parallel to MN. The height BC of this line above the vacuum line is $\frac{3}{4}''$; therefore, the absolute pressure at A or B is $\frac{3}{4} \times 40 = 30$ lb. The length $AB = l = 2.84''$, and the length L of the diagram is $3.6''$. The weight of a cubic foot of steam at 30 pounds pressure, absolute, is .074201 lb.

Substituting these values in the formula accompanying rule **184**,

$$Q = \frac{13{,}750\, l\, W}{P L} = \frac{13{,}750 \times 2.84 \times .074201}{35.36 \times 3.6}$$
$$= 22.76 \text{ lb. per I. H. P. per hour. Ans.}$$

(564) See Art. **1092.**

(565) (*a*) See Art. **1043.**

(*b*) The three principal uses are the following:

1. To find the I. H. P. of the engine.

2. To detect defects in valve setting, and to serve as a guide in setting the valves.

3. To determine, approximately, the steam consumption of the engine.

(566) (*a*) Work per stroke per square inch of piston =

$$\frac{3.47 \times 6}{12} \times 40 = 69.4 \text{ ft.-lb.}$$

Area of piston $= 16^2 \times .7854 = 201.0624$ sq. in.

Then, total work per stroke $= 69.4 \times 201.0624 = 13{,}953.73$ ft.-lb. Ans.

(*b*) 120 rev. per min. $= 120 \times 2 = 240$ strokes per min.

Then, H. P. $= \dfrac{13{,}953.73 \times 240}{33{,}000} = 101.48$ H. P. Ans.

(567) See Arts. **1098** and **1099.**

(568) (*a*) Using rule **180**,

$$\text{I. H. P.} = \frac{P L A N}{33{,}000} = \frac{47.1 \times \frac{18}{12} \times 11^2 \times .7854 \times 210 \times 2}{33{,}000} =$$
$$85.45 \text{ H. P. Ans.}$$

(*b*) $85.45 \times .83 = 70.92$ actual H. P. Ans.

(*c*) $85.45 - 70.92 = 14.53$ friction H. P. Ans.

(569) (*a*) Sixty pounds, gauge pressure, $= 60 + 14.7 = 74.7$ lb., absolute; 2 lb. above atmosphere $= 2 + 14.7 = 16.7$ lb., absolute.

Temperature of steam at 74.7 lb. pressure is, from the table,

$$306.526+\left(\frac{308.344-306.526}{2}\right)\times .7=307.162^\circ.$$

Temperature of steam at 16.7 lb. pressure is

$216.347+(219.452-216.347)\times .7=218.521^\circ$. Then,

$$T_1=307.162^\circ+460^\circ=767.162^\circ,$$

and $$T_2=218.521^\circ+460^\circ=678.521^\circ.$$

Thermal efficiency $=\frac{T_1-T_2}{T_1}=\frac{767.162-678.521}{767.162}=$ $.1155=11.55\%$. Ans.

(*b*) In this case, the absolute pressure of the entering steam is $90+14.7=104.7$ lb. per sq. in., and the pressure of the exhaust steam 3 lb. per sq. in., absolute.

The temperature corresponding to the former pressure is from the table $=331.169-\left(\frac{331.169-327.625}{5}\right)\times .3=$ 330.956°. The temperature corresponding to the latter pressure is 141.654°. Hence, T_1 is $330.956^\circ+460^\circ=790.956^\circ$, and T_2 is $141.654^\circ+460^\circ=601.654^\circ$.

Thermal efficiency $=\frac{790.956-601.654}{790.956}=.2393=23.93\%$. Ans.

(570) Area of high-pressure cylinder $=19^2\times .7854$ sq. in.

Area of low-pressure cylinder $=32^2\times .7854=804.25$ sq. in.

Ratio of area of high-pressure cylinder to area of low-pressure cylinder $=\frac{19^2\times .7854}{32^2\times .7854}=\frac{19^2}{32^2}=\frac{361}{1,024}$.

M. E. P. of high-pressure cylinder reduced to low-pressure cylinder $=52\times\frac{361}{1,024}=18.332$ lb. per sq. in.

Total M. E. P. reduced to low-pressure cylinder $=$

$$18+18.332=36.332 \text{ lb. per sq. in.}$$

(*a*) Substituting now, in rule **180**,

$$\text{I. H. P.}=\frac{PLAN}{33,000}=\frac{36.332\times\frac{30}{12}\times 804.25\times 120\times 2}{33,000}=531.27.$$ Ans.

(*b*) Since the stroke is the same for each cylinder, the ratio of the work done by the two cylinders is proportional to the ratio of the two M. E. P.'s reduced to the low-pressure cylinder. Hence, the ratio of the work done in the high-pressure cylinder to that done in the low-pressure cylinder is 18.332 : 18 = 1.0184. Ans.

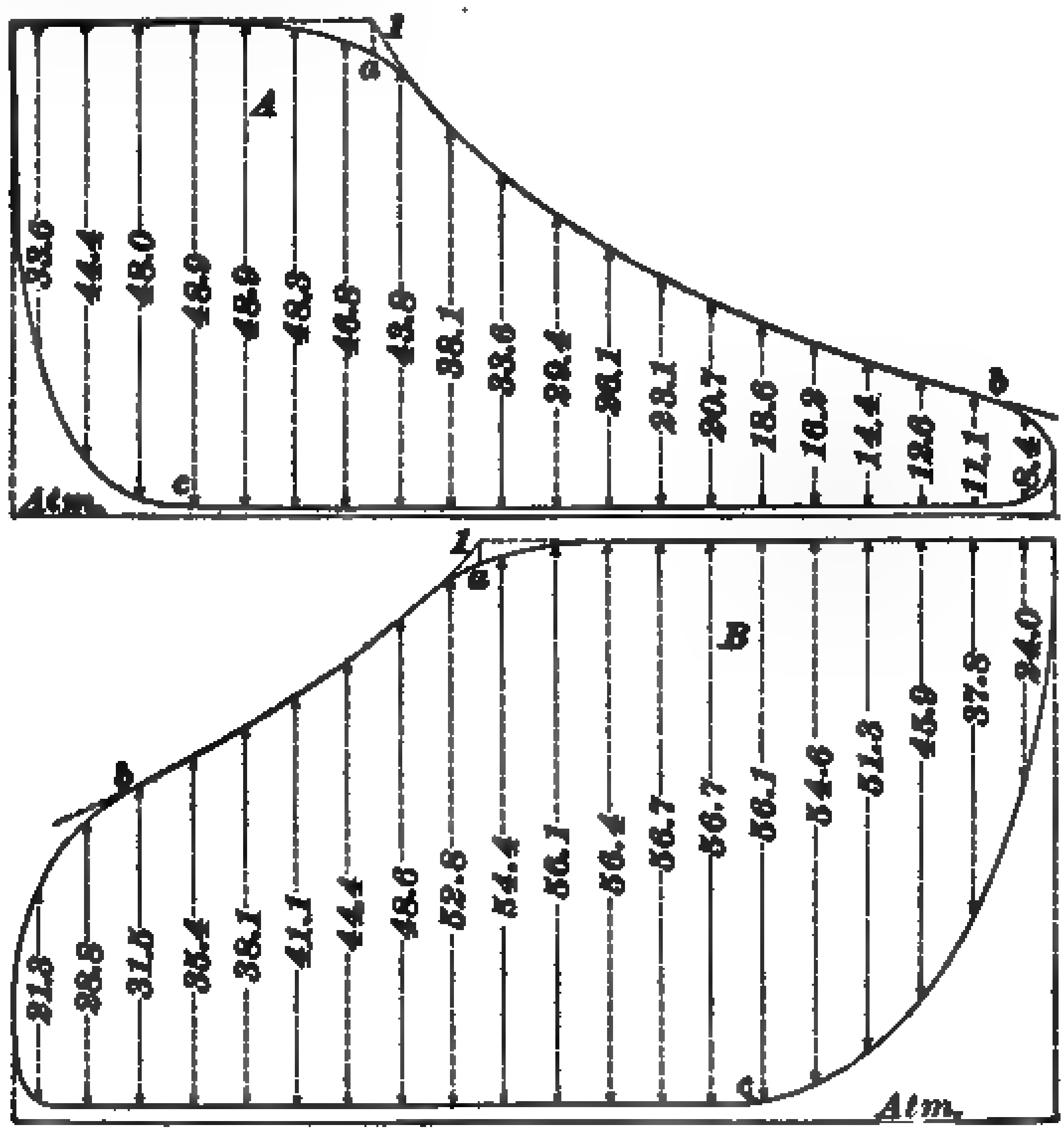


FIG. 21.

(571) (*a*) See Art. **1058.**

(*b*) 1. Decrease the angular advance.

2. Increase the angular advance.

3. Lower the boiler pressure or decrease the number of revolutions.

4. Raise the boiler pressure or increase the number of revolutions.

(572) (*a*) The points desired are shown in Fig. 21. To locate the point of cut-off, prolong the steam and expansion lines till they intersect as at *1*. From *1*, drop the perpendicular *1 a*, and *a* will be the point of cut-off.

The point of release may be located by prolonging the expansion curve, and noting the point where the actual curve departs from it, as shown at *b*. The point of compression *c* is easily located.

(*b*) The M. E. P.'s of the two diagrams are found as shown in the figure. The length of each diagram is divided into 20 equal parts, and ordinates are erected at the middle points of the divisions. The lengths of these ordinates multiplied by the scale of the spring used, in this case 1″ = 30 lb., added together and divided by 20 gives the required M. E. P. The M. E. P. of diagram *A* is found to be 30.75 lb. per sq. in., and the M. E. P. of diagram *B* is found to be 44.6 lb. per sq. in. Ans.

(573) See Arts. **1060** and **1061**.

(574) The total heat of steam at $4\frac{1}{2}$ pounds pressure is, from the steam tables,

$$\frac{1{,}128.641 + 1{,}131.462}{2} = 1{,}130.051 \text{ B. T. U.}$$

Now, using rule **185**,

$$W = \frac{H - t_2 + 32}{t_1 - t_2} = \frac{1{,}130.051 - 130 + 32}{130 - 55} = 13.76 \text{ lb.} \quad \text{Ans.}$$

(575) (*a*) Efficiency $= \frac{12.325}{15.36} = .8024 = 80.24\%$. Ans.

(*b*) Area of piston $= .7854 \times 9^2 = 63.617$ sq. in. Stroke $=$ 12″ = 1 foot.

Using rule **180**, I. H. P. $= \frac{PLAN}{33{,}000}$, or

$$P = \frac{\text{I. H. P.} \times 33{,}000}{LAN} = \frac{15.36 \times 33{,}000}{1 \times 63.617 \times 240 \times 2} = 16.6 \text{ lb.}$$

(576) To obtain an idea of the condensation of the steam in the cylinder. There is generally more condensation

at cut-off than at release, on account of reevaporation, and if the steam consumption be calculated from both points, the difference will tell, more or less approximately, the amount of condensation at cut-off.

(577) The scale of the spring should be, in general, not less than $\frac{1}{2}$ of the boiler pressure.

(*a*) $\frac{54}{2} = 27$; hence, a 30 spring should be used. Ans.

(*b*) $\frac{115}{2} = 57\frac{1}{2}$; hence, a 60 spring should be used. Ans.

(*c*) $1.834'' \times 40 = 73.36$ lb. per sq. in. Ans.

(578) Total heat of steam at an absolute pressure of 7 pounds is, from the table, 1,135.908. Using rule **185**,

$$W = \frac{H - t_1 + 32}{t_1 - t_2} =$$

$$\frac{1{,}135.908 - 148.66 + 32}{120 - 52} \times 906 = 13{,}580 \text{ lb.} \quad \text{Ans.}$$

(579) The areas of the three pistons are proportional to the squares of their respective diameters. Hence, the M. E. P. of the high-pressure cylinder reduced to the low-pressure cylinder is $72 \times \frac{27^2}{66^2} = 12.05$ lb. per sq. in. The M. E. P. of the intermediate cylinder reduced to the low-pressure cylinder is $40 \times \frac{42^2}{66^2} = 16.2$ lb. per sq. in.

The total M. E. P. reduced to the low-pressure cylinder is, consequently, $12.05 + 16.2 + 16.5 = 44.75$ lb. per sq. in.

Area of low-pressure piston $= .7854 \times 66^2 = 3{,}421.2$ sq. in.

(*a*) Using rule **180**,

$$\text{I. H. P.} = \frac{PLAN}{33{,}000} = \frac{44.75 \times 4 \times 3{,}421.2 \times 70 \times 2}{33{,}000} = 2{,}598 \text{ H. P.}$$

Ans.

(*b*) The work done in each cylinder is proportional to the M. E. P. of the cylinder reduced to the low-pressure cylinder. Hence, the percentage of total work done in the high-pressure

cylinder is $\frac{12.05}{44.75} = .269 = 26.9\%$. Ans. The percentage of work done in the intermediate cylinder is $\frac{16.2}{44.75} = .362 = 36.2\%$. Ans. Lastly, the percentage of the work done in the low-pressure cylinder is $\frac{16.5}{44.75} = .369 = 36.9\%$. Ans.

(580) Length of stroke = 24 inches = 2 feet. Area of piston = $18^2 \times .7854 = 254.47$ sq. in. Using rule **180**,

$$\text{I. H. P.} = \frac{PLAN}{33{,}000} = \frac{62.4 \times 2 \times 254.47 \times 175 \times 2}{33{,}000} = 336.825$$

horsepower. Ans.

(581) This example is worked like the previous examples where the horsepower is to be found from the diagrams. Dividing the length of the diagrams into 10 equal parts, and measuring the middle ordinates, their sum for diagram A is found to be 6.04″, and for B, 6.62″. The mean ordinate for $A = \frac{6.04}{10} = .604$, and for B, $\frac{6.62}{10} = .662$. Hence, the total

$$\text{M. E. P.} = \frac{.604 + .662}{2} \times 60 = 37.98 \text{ lb. per sq. in.}$$

$$\text{I. H. P.} = \frac{37.98 \times 1 \times 13^2 \times .7854 \times 300 \times 2}{33{,}000} = 91.66 \text{ H. P.} \quad \text{Ans.}$$

(582) Drawing a line parallel to the atmospheric line, and at a distance from the vacuum line corresponding to a pressure of 40 lb., absolute, the length included between the bounding lines of diagram A is 1.46″, and the length between the bounding lines of diagram B is 1.72″. The length of both diagrams is 2.6″. The weight of a cubic foot of steam, at an absolute pressure of 40 lb., is .097231 lb. The M. E. P. for card $A = .604 \times 60 = 36.24$ lb., and for card B, $.662 \times 60 = 39.72$ lb. (See Example 581).

Using rule **184**, we have, for card A,

$$Q = \frac{13{,}750 \times 1.46 \times .097231}{36.24 \times 2.6} = 20.716 \text{ lb.},$$

and for card B,

$$Q = \frac{13{,}750 \times 1.72 \times .097231}{39.72 \times 2.6} = 22.267 \text{ lb.}$$

Taking the average of both cards,

$$Q = \frac{20.716 + 22.267}{2} = 21.49 \text{ lb. per I. H. P. per hour. Ans.}$$

(583) By rule **170**, Art. **994**, the real cut-off in small cylinder is

$$k = \frac{s + i}{1 + i} = \frac{.375 + .08}{1.08} = \frac{.455}{1.08}.$$

The ratio of expansion is, therefore,

$$e = \frac{1}{k} = \frac{1.08}{.455} = 2.374.$$

The volumes of the large and small cylinders are proportional to the square of their diameters.

$$V : v :: 68^2 : 30^2,$$

$$\text{or } \frac{V}{v} = \frac{68^2}{30^2}.$$

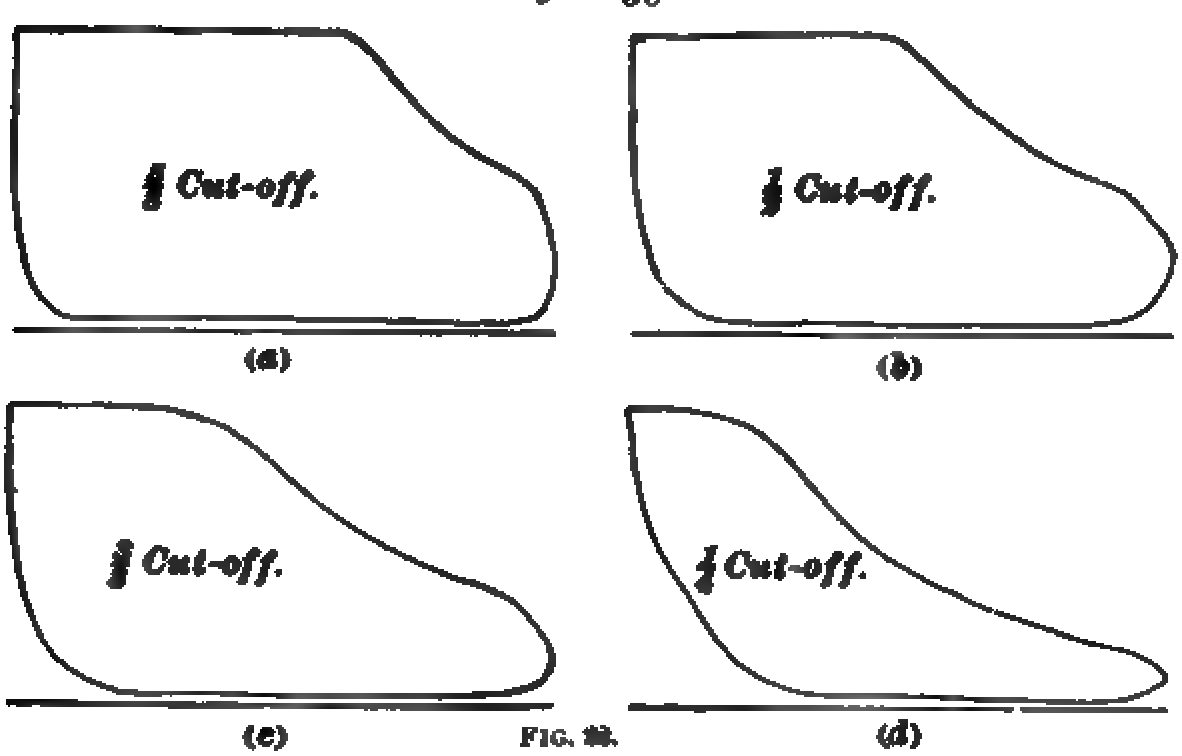


Fig. [illegible].

By rule **186**, the total ratio of expansion is

$$E = \frac{eV}{v} = \frac{2.374 \times 68^2}{30^2} = 12.2.$$

(584) (*a*) $143 + 460 = 603°$; $256 + 460 = 716°$; $47.2 + 460 = 507.2°$.

(*b*) $785 - 460 = 325°$; $492 - 460 = 32°$; $443 - 460 = -17°$; that is, 17° below 0.

(585) (*a*) (*b*) As the cut-off becomes less and less, the compression is increased. The diagrams would be about as shown in Fig. 22.

The increase of compression is the distinguishing feature of the diagrams.

(*c*) See Art. **1056.**

(586) Length of stroke : length of card :: length of lever : distance from point;

$$\text{or, } 32 : 3\tfrac{1}{4} :: 8 \times 12 : x;$$

$$\text{or, } \quad x = \frac{8 \times 12 \times 3\frac{1}{4}}{32} = 9\tfrac{3}{4} \text{ in.} \quad \text{Ans.}$$

(587) Assume the engine to be 16″ × 20″, and to make 180 R. P. M. Suppose the M. E. P. to be 42.7 lb. per sq. in. Using rule **169**, Art. **937**,

$$\frac{144\,p\,V}{33{,}000} \times 180 \times 2 = \frac{144 \times 42.7 \times \dfrac{16^2 \times .7854 \times 20}{1.728}}{33{,}000} \times 180 \times 2 = 156.1 \text{ H. P.} \quad \text{Ans.}$$

Using rule **180**,

$$\frac{PLAN}{33{,}000} = \frac{42.7 \times \frac{20}{12} \times 16^2 \times .7854 \times 180 \times 2}{33{,}000} = 156.1 \text{ H. P.} \quad \text{Ans.}$$

The student must assume different values from those given above when answering this question.

(588) $(30 - 23.9) \times \dfrac{14.7}{30} = 3$ lb. = absolute pressure in the condenser.

The total heat of a pound of steam at a pressure of 3 lb. above vacuum is, from the steam table, 1,125.144 B. T. U. Applying rule **185**, $W = \dfrac{H - t_2 + 32}{t_1 - t_2} = \dfrac{1{,}125.144 - 124 + 32}{105 - 52} =$ 19.49 lb. of water required per pound of steam. Hence, total weight of water required per minute =

$$\frac{19.49 \times 271.6 \times 27.8}{60} = 2{,}452.6 \text{ lb.} \quad \text{Ans.}$$

(589) See Art. **1094.**

(590) See Arts. **1119** to **1121.**

(591) See Arts. **1123** to **1125.**

(592) See Art. **1122.**

(593) See Art. **1125.**

(594) See Art. **1127.**

(595) See Art. **1128.**

(596) See Art. **1133.**

(597) See Art. **1115.**

(598) See Art. **1134.**

(599) See Art. **1142.**

(600) See Art. **1143.**

(601) See Art. **1146.**

(602) See Art. **1151.**

With a right-handed propeller, when in the go-ahead motion, the pressure will be on the port guide; in the backing motion, on the starboard guide. With a left-handed propeller and in the go-ahead motion, the pressure will be on the starboard guide; in the backing motion, on the port guide.

(603) Find the clearance volume first. It is $\frac{8 \times 24^2 \times .7854 \times 48}{100} = 1{,}737.14$ cu. in., nearly. The distance the piston moves up to the time of cut-off is $\frac{48 \times 5}{8} =$ 30 inches. The volume displaced is, therefore, $24^2 \times .7854 \times 30 = 13{,}571.7$ cu. in. Hence, the total volume of steam per stroke is $13{,}571.7 + 1{,}737.14 = 15{,}308.84$ cu. in. Since the plunger feed-pump makes one delivery stroke for every two strokes of the engine, the volume of steam per stroke of the pump is $15{,}308.84 \times 2 = 30{,}617.68$ cu. in. From the steam tables, the ratio between steam at 170 lb. pressure, absolute, and water is 164.3. The theoretical quantity of water for

each delivery stroke of the pump is, therefore, 30,617.68 ÷ 164.3 = 186.35 cu. in. Since two pumps are used, each should deliver 186.35 × 3 = 559 cu. in., nearly. See Art. **1153.** The area of the plunger is, then, 559 ÷ 24 = 23.29 sq. in. The diameter corresponding to this area is $5\frac{1}{2}$ in., nearly.

(604) The net feed-water per delivery stroke of the pump is the same as in Example 603, that is, 186.35 cubic inches. The gross feed-water is then $\frac{2}{2-1} \times 186.35 = 372.7$ cu. in. Making the pump large enough to deliver twice this quantity, we have 745.4 cubic inches to deliver. The area of the plunger is, then, 745.4 ÷ 24 = 31.06 sq. in. The corresponding diameter is $6\frac{5}{16}$ inches, nearly.

(605) See Arts. **1139** to **1141.**

THE MACHINERY OF WESTERN RIVER STEAMBOATS.

(1) It will run *under*. See Arts. **8** and **31**.

(2) A fixed cut-off engine is one in which the cut-off can not be changed. A variable cut-off engine is one in which it may be changed at will.

(3) See Arts. **27, 28,** and **29**.

(4) See Arts. **6** and **7**.

(5) See Art. **10**.

(6) See Art. **10**.

(7) See Art. **13**.

(8) See Art. **18**.

(9) See Art. **16**.

(10) See Art. **16**.

(11) See Art. **20**.

(12) See Art. **18**.

(13) See Art. **21**.

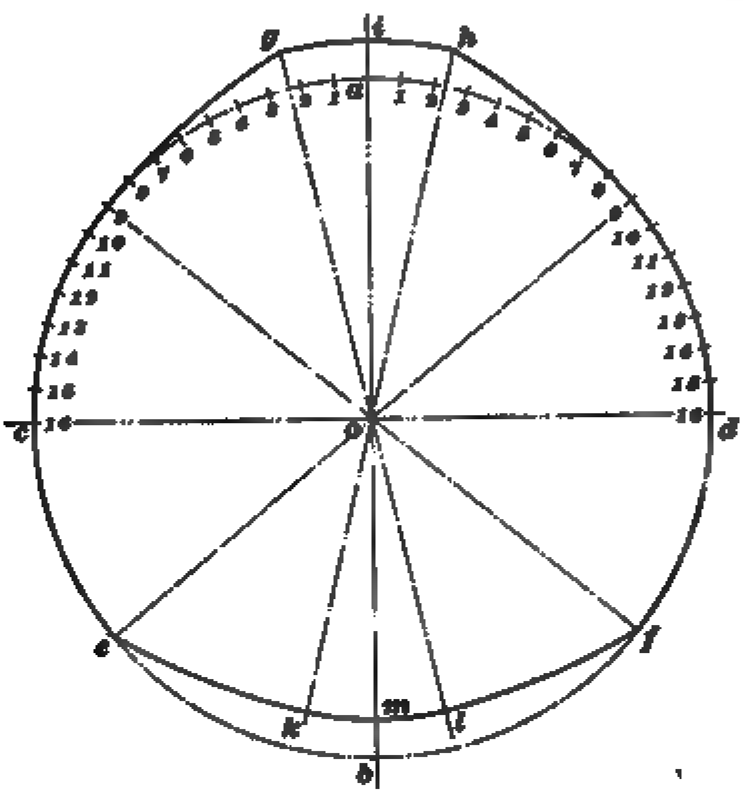
(14) See Art. **37**.

(15) See Art. **35**.

(16) See Arts. **36, 40,** and **46.**

(17) See Art. **43.**

(18) The construction of the cam is snown in the figure drawn to a scale of $1\frac{1}{2}$ inches = 1 ft. Draw the circle *a d b c* 18 inches in diameter. Draw the vertical center line *a b* and horizontal center line *c d*. Divide each of the quadrants *a c* and *a d* into 16 equal parts, as explained in Art. **45.** Point off 9 divisions to the right and left of *a*. Then through the center *o* draw the straight lines *9 f* and *9 e*. Next, lay off the distance *a i* equal to 1 inch, the required throw. With *o* as a center and a radius *o i*, describe an arc *g i h*. Then, with *e* and *f* as centers and a radius equal to the size of the cam, describe the arcs *9 g* and *9 h*. Through *g* and *h* and *o* draw the straight lines *g l* and *h k*. Now, with *g* and *h* as centers and a radius equal to the size of the cam, describe the arcs *e k* and *l f*. Then, with *o* as a center and a radius *o k*, describe the arc *k m l*. *b m* must now be exactly equal to *a i*. The required cam outline is *g i h d f l m k e c g*.



Scale $1\frac{1}{2}$ inches=1 ft.

(19) See Art. **48.**

(20) The point at which the steam valves open is $\frac{55}{60} = \frac{11}{12}$ of the stroke. Then, according to Art. **51,** the cut-off for which the cam is to be designed is $(1 - \frac{11}{12}) + \frac{1}{2} = \frac{7}{12}$. Ans.

(21) See Art. **53.**

(22) See Art. **57.**

(23) See Art. **57.**

(24) See Art. **62.**

(25) See Art. **66.**

(26) See Art. **68.**

(27) See Art. **73.**

RECENT DEVELOPMENTS IN MARINE ENGINEERING.

(1) See Art. **5.**

(2) See Art. **6.**

(3) See Art. **2.**

(4) Heat units abstracted per day $= 90{,}000 \times 24 = 2{,}160{,}000$ B. T. U.

Then by the rule given in Art. **9**,

$$F = \frac{2{,}160{,}000}{288{,}000} = 7.5 \text{ tons.} \quad \text{Ans.}$$

(5) 12 tons $= 2{,}000 \times 12 = 24{,}000$ lb. Then, according to Art. **11**, the commercial efficiency is $24{,}000 \div 2{,}800 = 8.57$ lb. of ice melted per pound of coal burned.

(6) See Art. **15.**

(7) See Art. **27.**

(8) See Arts. **35, 36,** and **41.**

(9) See Arts. **43** and **44.**

(10) See Art. **45.**

(11) See Art. **46.**

(12) See Arts. **53** and **54.**

(13) See Art. **56.**

(14) See Art. **57.**

(15) See Art. **67.**

(16) See Art. **64.**

(17) See Art. **68.**

(18) See Arts. **73** and **74.**

(19) See Art. **79.**

(20) See Art. **90.**

DYNAMOS AND MOTORS.

(PART 1.)

(1) The end b; because, in looking at that end, the current circulates around the helix in an opposite direction to the hands of a watch. (Art. **29**.)

(2) (*a*) Negative. (Art. **7.**)
(*b*) Negative. (Art. **7.**)
(*c*) Positive. (Art. **7.**)

(3) By formula **6,** $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total generated E. M. F. in volts, and R is the total resistance in ohms of the circuit. In this example, $E = 20$ volts and $R = 30 + 80 = 110$ ohms; hence, $C = \frac{E}{R} = \frac{20}{110} = .1818$ ampere. Ans.

(4) Let A represent the first branch and B the second; then, $r_1 = 16.2$ ohms, $r_2 = 14.1$ ohms, and $C = 6.37$ amperes.

The current c_1 in branch A is found by using formula **10**; substituting, gives $c_1 = \frac{C r_2}{r_1 + r_2} = \frac{6.37 \times 14.1}{16.2 + 14.1} = 2.9643$ amperes. Ans.

The current c_2 in branch B is found by using formula **11**; substituting, gives

$$c_2 = \frac{C r_1}{r_1 + r_2} = \frac{6.37 \times 16.2}{16.2 + 14.1} = 3.4057 \text{ amperes. Ans.}$$

(5) From formula **23,** $W = \text{H. P.} \times 746$, where H. P. is the horsepower and W is the power in watts. In this example, H. P. = 2.33 horsepower; hence, $W = \text{H. P.} \times 746 = 2.33 \times 746 = 1{,}738.18$ watts. Ans.

(6) (*a*) From Art. **64** and formula **6,** $C = \frac{E'}{R'}$, where C is the current in amperes, E' is the difference of potential in volts between two points, and R' is the resistance in ohms between them. In this example, $E' = 58.4$ volts and $R' = 2.3$ ohms; hence, $C = \frac{E'}{R'} = \frac{58.4}{2.3} = 25.3913$ amperes. Ans.

(*b*) From formula **21,** $W = \frac{E^2}{R}$, where W is the power in watts, E is the E. M. F., or difference of potential in volts, and R is the resistance in ohms. In this example, $E = 58.4$ volts and $R = 2.3$ ohms; hence,

$$W = \frac{E^2}{R} = \frac{58.4^2}{2.3} = \frac{3{,}410.56}{2.3} = 1{,}482.8521 \text{ watts. Ans.}$$

(*c*) By formula **22,** H. P. $= \frac{W}{746}$; by formula **21,** $W = \frac{E^2}{R}$; therefore (see Art. **81**), H. P. $= \frac{E^2}{746\,R}$, where H. P. is the horsepower, E the E. M. F., or difference of potential in volts, and R the resistance in ohms.

Hence, H. P. $= \frac{58.4^2}{746 \times 2.3} = \frac{3{,}410.56}{1{,}715.8} = 1\ 9877$ horsepower. Ans.

(7) Platinum, as it follows zinc in the list (Art. **13**).

(8) Towards the *east* (Art. **26**).

(9) By formula **8,** $E = C\,R$, where E is the total E. M. F. in volts developed in a closed circuit, C is the current in amperes flowing, and R is the total resistance in ohms of the circuit. In this example, $C = .75$ ampere and $R = 17.2 + 8.2 + 11.3 = 36.7$ ohms; hence, $E = C\,R = .75 \times 36.7 = 27.525$ volts, the total E. M. F. developed in the battery.

By derivation from formula **8,** $E' = C\,R'$, where E' is the difference of potential in volts between two points, C is the current in amperes flowing, and R' is the resistance in ohms between the two points.

Between a and b, $R' = 11.3$ ohms and $C = .75$ ampere; hence, $E' = C R' = .75 \times 11.3 = 8.475$ volts. Ans.

Between b and c, $R' = 8.2$ ohms and $C = .75$ ampere; hence, $E' = C R' = .75 \times 8.2 = 6.15$ volts. Ans.

Between a and c, the difference of potential is the difference of potential between a and b plus that between b and c, which is $6.15 + 8.475 = 14.625$ volts. Or, since the difference of potential between a and c is the available E. M. F. of the battery, when a current of .75 ampere is flowing, it can be calculated by using formula **9**, $E' = E - C r_i$, where E' is the available E. M. F., E is the total E. M. F. developed in the battery, C is the current flowing, and r_i is the internal resistance of the battery. In this case, $E = 27.525$ volts, $C = .75$ ampere, and $r_i = 17.2$ ohms; therefore, $E' = E - C r_i = 27.525 - (.75 \times 17.2) = 14.625$ volts. Ans.

(10) The sectional area of a wire .2 in. in diameter is $.7854 \times .2 \times .2 = .031416$ sq. in., or, nearly, .0314 sq. in.

Reduce the specific resistance in microhms to the resistance in ohms by dividing by 1,000,000 (Art. **44**), which gives $\frac{.5921}{1,000,000} = .0000005921$ ohm; or, in other words, the resistance of a piece of silver one inch long, and whose sectional area is one square inch, is .0000005921 ohm. Next, from this resistance and length calculate the resistance of 1,000 feet of the silver with a sectional area of 1 sq. in., by using formula **1**, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance, r_2 is the resistance after the length of the conductor is changed, l_1 is the original length of the conductor, and l_2 is the changed length. In this example, $r_1 = .0000005921$ ohm, $l_1 = 1$ inch, and $l_2 = 1,000$ feet, or 12,000 inches. Hence, by substituting, $r_2 = \frac{r_1 l_2}{l_1} = .0000005921 \times \frac{12,000}{1} = .0071052$ ohm; that is, the resistance of 1,000 feet of silver having a sectional area of 1 sq. in. is .0071052 ohm. From this result calculate the resistance of 1,000 feet of the silver when its sectional area is .0314 sq. in., by using

formula **2**, $r_2 = \frac{r_1 a_1}{a_2}$, where r_1 is the original resistance, r_2 is the resistance after the sectional area has been changed, a_1 is the original area, and a_2 is the changed sectional area. At this stage of the example, $r_1 = .0071052$ ohm, $a_1 = 1$ sq. in., and $a_2 = .0314$ sq. in. Hence,

$$r_2 = \frac{.0071052 \times 1}{.0314} = .2262 \text{ ohm. Ans.}$$

(11) By formula **8**, $E = C R$, where E is the total E. M. F. in volts developed in a closed circuit, C is the current in amperes flowing, and R is the total resistance in ohms of the circuit. In this example, $C = .127$ ampere and $R = 36.2 + 21.7 = 57.9$ ohms. Hence, by substituting, $E = C R = .127 \times 57.9 = 7.3533$ volts. Ans.

(12) By formula **14**, $Q = C t$, where Q is the quantity of electricity in coulombs which passes through a circuit, C is the current in amperes flowing in that circuit, and t is the time in seconds during which the current flows. In this example, $C = 8.32$ amperes and $t = 2.25 \times 60 \times 60 = 8{,}100$ seconds. Hence, by substituting, $Q = C t = 8.32 \times 8{,}100 = 67{,}392$ coulombs. Ans.

(13) By formula **19**, $W = C E$, where W is the power in watts, E is the E. M. F. in volts, and C is the current in amperes. In this example, $E = 112.5$ volts and $C = 12.2$ amperes. Hence, by substituting, $W = C E = 12.2 \times 112.5 = 1{,}372.5$ watts. Ans.

(14) By formula **12**, the joint resistance of a derived circuit of two branches in parallel $R'' = \frac{r_1 r_2}{r_1 + r_2}$. In this case, $r_1 = 2.4$ and $r_2 = 987.3$; then their joint resistance in parallel $R'' = \frac{2.4 \times 987.3}{2.4 + 987.3} = \frac{2{,}369.52}{989.7} = 2.3941$ ohms. Ans.

(15) By formula **4**, $r_2 = r_1 (1 + t k)$, where r_1 is the original resistance of a conductor, r_2 is the resistance after a rise in temperature, k is the temperature coefficient, and t is the rise of temperature in degrees F. In this example,

$r_1 = 43.2$ ohms, $t = 85 - 60 = 25°$ F., and $k = .002155$, from Table 1. Hence, by substituting, $r_2 = r\ (1 + t\,k) = 43.2\ (1 + 25 \times .002155) = 43.2 \times 1.053875 = 45.5274$ ohms. Ans.

(16) By formula **13**, the joint resistance of three conductors in parallel $R''' = \dfrac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}$, where r_1, r_2, and r_3 are the separate resistances of the three conductors, respectively. In this example, let $r_1 = 37$ ohms, the resistance of A; $r_2 = 45$ ohms, the resistance of B; and $r_3 = 72$ ohms, the resistance of C. Substituting, we have

$$\frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} = \frac{37 \times 45 \times 72}{45 \times 72 + 37 \times 72 + 37 \times 45} = \frac{119{,}880}{7{,}569} =$$

15.8383 ohms, the joint resistance of the three conductors A, B, and C connected in parallel. Ans.

(17) By formula **8**, $E = C\,R$, where E is the total E. M. F. in volts developed within a closed circuit, C is the current in amperes, and R is the total resistance in ohms of the circuit. In this example, $C = 2.73$ amperes and $R = 49.3$ ohms; hence, by substituting, $E = C\,R = 2.73 \times 49.3 = 134.589$ volts. Ans.

(18) From Art. **43**, the joint resistance of several conductors connected in series is equal to the sum of their separate resistances; hence, in this example, the joint resistance of the four conductors A, B, C, and D, in series, is $3 + 19 + 72 + 111 = 205$ ohms. Ans.

(19) (*a*) By formula **7**, $R = \dfrac{E}{C}$, where R is the total resistance in ohms of a closed circuit, E is the total E.M.F. in volts developed, and C is the current in amperes flowing in the circuit. In this example, $E = 28.2$ volts and $C = 5.2$ amperes; hence, $R = \dfrac{28.2}{5.2} = 5.423$ ohms. Ans.

(*b*) The total resistance of a closed circuit, Art. **60**, is equal to the sum of the internal and external resistances. Since in this example the external resistance is 7 times the

internal, and their sum is 5.423, let $\frac{1}{8}$ of the total resistance represent the internal, and, therefore, $\frac{7}{8}$ of the total resistance represents the external resistance. Hence, $\frac{1}{8} \times 5.423 = .677875$ ohm, the internal resistance. Ans.

And $\frac{7}{8} \times 5.423 = 4.745125$ ohms, the external resistance. Ans.

(20) We here use formula **16**, $J = C^2 R t$, where J is the work in joules, C is the current in amperes, R is the resistance in ohms, and t is the time in seconds. In this case, $C = 14.2$ amperes, $R = 8$ ohms, $t = 4{,}500$ seconds. Then, the work done $= 14.2 \times 14.2 \times 8 \times 4{,}500 = 7{,}259{,}040$ joules. Ans.

(21) By formula **5**, $r_2 = \frac{r_1}{1 + t k}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its temperature has fallen, t is the fall of temperature in degrees F., and k is the temperature coefficient. In this example, $r_1 = 214$ ohms, $t = 82 - 50 = 32°$ F., and $k = .002094$, from Table 1. Hence,

$$r_2 = \frac{r_1}{1 + t k} = \frac{214}{1 + 32 \times .002094} = \frac{214}{1.067008} = 200.5608 \text{ ohms.}$$ Ans.

(22) From Art. **75**, the separate resistance of any branch of a derived circuit is equal to the difference of potential between where all the branches divide and where they unite, divided by the current in that branch.

Hence, the separate resistance of branch A is $\frac{11.6}{6.7} = 1.7313$ ohms. Ans.

The separate resistance of branch B is $\frac{11.6}{4.9} = 2.3673$ ohms. Ans.

(23) By formula **7**, $R = \frac{E}{C}$, where R is the total resistance in ohms of a closed circuit, E is the total E. M. F. in volts developed in the circuit, and C is the current in amperes flowing in the circuit. In this example, $E = 22.4$ volts and $C = .43$ ampere; hence, $R = \frac{E}{C} = \frac{22.4}{.43} = 52.093$

ohms, the total resistance of the circuit. Since the total resistance of a closed circuit is equal to the sum of the external and internal resistances, the external resistance must be the difference between the total resistance and the internal resistance. Hence, the external resistance $= 52.093 - 13.4 = 38.693$ ohms. Ans.

(24) By transposition of terms in formula **14**, $C = \frac{Q}{t}$, where C is the current in amperes, Q is the quantity of electricity in coulombs, and t is the time in seconds. In this example, $Q = 368{,}422$ coulombs and $t = 4.5 \times 60 \times 60 = 16{,}200$ seconds; hence, $C = \frac{Q}{t} = \frac{368{,}422}{16{,}200} = 22.7421$ amperes. Ans.

(25) By formula **16**, $J = C^2 R t$, where J is the work done in joules, C is the current in amperes, R is the resistance in ohms, and t is the time in seconds. In this example, $C = 2.4$ amperes, $R = 45$ ohms, and $t = 3{,}000$ seconds. Then the electrical work done $= 2.4 \times 2.4 \times 45 \times 3{,}000 = 777{,}600$ joules. By formula **18**, the mechanical work done $=$ F. P. $= .7373\ J = .7373 \times 777{,}600 = 573{,}324.48$ foot-pounds. Ans.

(26) By formula **22**, H. P. $= \frac{W}{746}$; by formula **19**, $W = C E$; therefore (see Art. **81**), H. P. $= \frac{E\,C}{746}$, where H. P. is the horsepower, E is the E. M. F. in volts, and C is the current in amperes. In this example, $E = 525$ volts and $C = 12.5$ amperes; hence,

$$\text{H. P.} = \frac{E\,C}{746} = \frac{525 \times 12.5}{746} = 8.7969 \text{ horsepower. Ans.}$$

(27) (*a*) By formula **20**, $W = C^2 R$, where W is the power in watts, C is the current in amperes, and R is the resistance in ohms. In this example, $C = 110$ amperes and $R = 4.2$ ohms; hence, $W = C^2 R = 110^2 \times 4.2 = 50{,}820$ watts. Ans.

(*b*) By formula **22**, H. P. $= \frac{W}{746}$, where H. P. is the

horsepower and W is the power in watts. In this example, $W = 50,820$ watts; hence,

$$\text{H. P.} = \frac{W}{746} = \frac{50,820}{746} = 68.1233 \text{ horsepower. Ans.}$$

(28) The diagram, Fig. 1, shows the connections of the battery and galvanometer circuits to the circular type of resistance-box for measuring unknown resistances by the Wheatstone-bridge method. The upper balance arm (Art. **53**) of the bridge includes the resistance coils from c to a, the lower balance arm includes the coils from a to d, and the adjustable arm includes the coils from d to b. One pole of the battery B is connected to the junction of the two balance arms, the other to the junction of the adjustable arm and the unknown resistance X. One terminal of the galvanometer G is connected to the junction of the lower balance arm and the adjustable arm, the other to the junction of the upper balance arm and the unknown resistance.

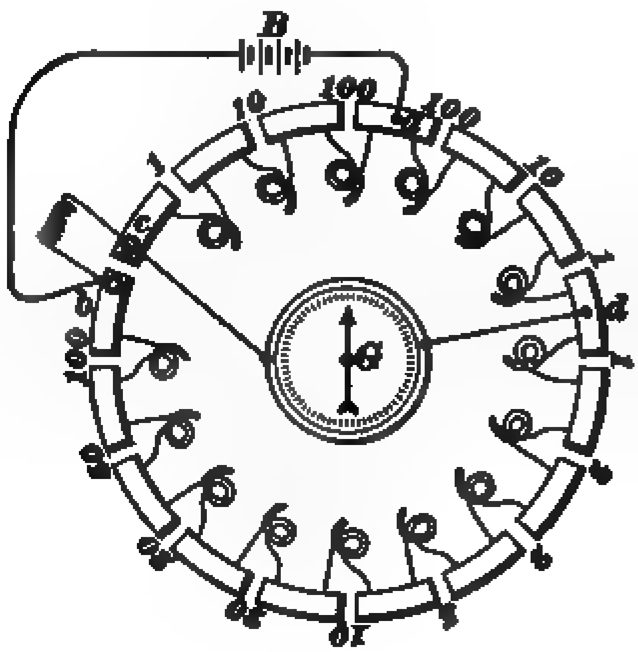


Fig. 1.

(29) By formula **1**, the changed resistance for variation in length $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance, l_1 is the original length, and l_2 is the changed length. In this case, $r_1 = 1$ ohm, $l_1 = 1,000$ feet, and $l_2 = 2,000$ feet. Then, the changed resistance $r_2 = \frac{1 \times 2,000}{1,000} = 2$ ohms. The next operation is to determine the resistance of the wire when its sectional area is changed. A round wire .1" in diameter has a sectional area of $.1^2 \times .7854 = .007854$ sq. in., and a square wire .1" on a side has a sectional area of $.1 \times .1 = .01$ sq. in. By formula **2**, $r_2 = \frac{r_1 a_1}{a_2}$, where r_1 is

the original resistance of a conductor, r_2 is the resistance after its sectional area is changed, a_1 is the original sectional area, and a_2 is the changed sectional area. At this stage of the example, $r_1 = 2$ ohms, $a_1 = .007854$ sq. in., and $a_2 = .01$ sq. in. Hence, $r_2 = \frac{r_1 a_1}{a_2} = \frac{2 \times .007854}{.01} = 1.5708$ ohms. Ans.

(30) By formula **22**, H. P. $= \frac{W}{746}$, where H. P. is the horsepower and W is the power in watts. In this example, $W = 54{,}200$ watts; hence, H. P. $= \frac{W}{746} = \frac{54{,}200}{746} = 72.6541$ horsepower. Ans.

(31) The sectional area of a round column .04 in. in diameter is $.04^2 \times .7854 = .00125664$ sq. in., or .001257 sq. in., nearly.

Reduce the specific resistance in microhms to the resistance in ohms by dividing by 1,000,000, Art. **44**, which gives $\frac{37.15}{1{,}000{,}000} = .00003715$ ohm; or, in other words, the resistance of a quantity of mercury 1 in. long, and whose sectional area is 1 sq. in., is .00003715 ohm. Next, from this resistance and length, calculate the resistance of a column of mercury 72.3″ high, with a sectional area of 1 sq. in. by using formula **1**, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its length has been changed, l_1 is its original length, and l_2 is its changed length. In this example, $r_1 = .00003715$ ohm, $l_1 = 1''$, and $l_2 = 72.3$ inches. Hence, $r_2 = \frac{r_1 l_2}{l_1} = \frac{.00003715 \times 72.3}{1} = .002685945$, or .002686 ohm, nearly; or, in other words, the resistance of a column of mercury 72.3″ high, having a sectional area of 1 sq. in., is .002686 ohm. From this result calculate the resistance of the column when its sectional area is .001257 sq. in., by using formula **2**, $r_2 = \frac{r_1 a_1}{a_2}$, where r_1 is the original resistance,

r_2 is the resistance after the sectional area has been changed, a_1 is the original sectional area, and a_2 is the changed sectional area. At this stage of the example, $r_1 = .002686$ ohm, $a_1 = 1$ sq. in., and $a_2 = .001257$ sq. in. Hence,

$$r_2 = \frac{r_1 a_1}{a_2} = \frac{.002686 \times 1}{.001257} = 2.1368 \text{ ohms. Ans.}$$

(32) By formula **6**, $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total E. M. F. in volts generated, and R is the total resistance in ohms of the circuit. Since the total resistance of a closed circuit is the sum of the external and internal circuits, $R = 33 + 30 = 63$ ohms and $E = 45$ volts; hence, $C = \frac{E}{R} = \frac{45}{63} = .7143$ ampere. Ans.

(33) In Fig. 5, question 33, the reading of the voltmeter gives the total E. M. F. of the battery. Hence, after the connections are made, as shown in Fig. 6, question 33, there is a closed circuit in which the total E. M. F. developed is 24.4 volts, and through which a current of .8 ampere is flowing. By formula **7**, $R = \frac{E}{C}$, where R is the total resistance in ohms of a closed circuit, E is the total E. M. F. in volts developed, and C is the current in amperes flowing. In this example, $E = 24.4$ volts and $C = .8$ ampere; hence, $R = \frac{E}{C} = \frac{24.4}{.8} = 30.5$ ohms, the total resistance of the circuit.

From the reading of the voltmeter, after the connections are made as shown in Fig. 6, it will be seen that when a current of .8 ampere flows through the external resistance from b to a through R, there is a drop or loss of potential of 18 volts. By Art. **64** and formula **7**, $R' = \frac{E'}{C}$, where R' is the resistance of a conductor, E' the drop, or loss, of potential in that conductor, and C is the current in amperes

flowing through it. In this case, $E' = 18$ volts and $C = .8$ ampere; hence, $R' = \frac{E'}{C} = \frac{18}{.8} = 22.5$ ohms, the resistance of the external circuit from b to a through the resistance R. Ans.

Since the total resistance of a closed circuit is the sum of the external and internal resistances, Art. **60**, the internal resistance must be the difference between the total and external resistances. Hence, $30.5 - 22.5 = 8$ ohms, the internal resistance of the battery B. Ans.

(34) By formula **19**, $W = C E$, where W is the power in watts, E is the E. M. F., or difference of potential in volts, and C is the current in amperes. In this example, $E = 510$ volts and $C = 24.3$ amperes; hence, $W = 510 \times 24.3 = 12{,}393$ watts. Ans.

(35) Referring to Art. **56**, the total E. M. F. developed by connecting several cells in series is equal to the E. M. F. of one cell multiplied by the number of cells; hence, the E. M. F. of one of the groups of 6 cells is $6 \times 1.5 = 9$ volts. In the same article it is stated that connecting cells in multiple, or parallel, does not change the E. M. F. between the main conductors. In this case, each group of six cells can be considered as one large cell developing an E. M. F. of 9 volts, and, consequently, the E. M. F. of the four groups connected in multiple, or parallel, is 9 volts, which would be the E. M. F. indicated by a voltmeter connected to the main conductors c and c', as shown in Fig. 7, question 35. Ans.

(36) By formula **22**, H. P. $= \frac{W}{746}$; by formula **19**, $W = C E$; therefore (see Art. **81**), H. P. $= \frac{E\,C}{746}$, where H. P. is the horsepower, E is the E. M. F. in volts, and C is the current in amperes. In this example, $E = 250$ volts and $C = 65.7$ amperes; hence, H. P. $= \frac{E\,C}{746} = \frac{250 \times 65.7}{746} = \frac{16{,}425}{746} = 22.0174$ horsepower. Ans.

(37) In Art. **26** it is stated that when a compass is placed under a conductor in which an electric current is flowing from the south to the north, the north pole of the compass needle tends to point towards the west, and if the direction of the current in the conductor is reversed, the north pole will point towards the east. Since, in this example, the north pole of the needle tends to point towards the east, the current must be flowing *from* the north *to* the south.

(38) End *a*, since (Art. **29**), in looking at the face of the end *a*, the current circulates around the core in the same direction as the movement of the hands of a watch.

(39) Attract one another; since (Art. **7**) a positive charge is developed upon the ivory when rubbed with silk and a negative charge upon sealing-wax when rubbed with fur; and from Art. **6**, electrified bodies with dissimilar charges are mutually attractive.

(40) The exposed end of the iron; since, from Art. **13**, the iron forms the positive element of the cell, and, from Art. **12**, the pole, or electrode, attached to the exposed end of a voltaic element is always of opposite sign to the element itself.

(41) From Art. **21**, iron and its alloys, nickel, cobalt, manganese, oxygen, cerium, and chromium.

(42) Towards the south pole, since, from Art. **20**, unlike poles attract one another.

(43) Towards the north pole, since, from Art. **20**, unlike poles attract one another.

(44) *From* the north *to* the south, since, from Art. **26**, the north pole of a compass needle tends to point towards the east when the compass is placed over a conductor in which the current is flowing from the south to the north; and by reversing the direction of the current in the conductor, the north pole of the needle tends to point towards the west and the south pole towards the east.

(45) The current should enter the wire at end *b*; since (Art. **29**), in looking at the face of the south pole

of the magnet, the current circulates around the core in the direction of the motion of the hands of a watch.

(46) By formula **1**, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its length has been changed, l_1 is the original length, and l_2 is its changed length. In this example, $r_1 = 100.8$ ohms, $l_1 = (112 \times 12) + 6 = 1{,}350$ inches, and $l_2 = 11.7$ inches. Hence,

$$r_2 = \frac{r_1 l_2}{l_1} = \frac{100.8 \times 11.7}{1{,}350} = .8736 \text{ ohm.} \quad \text{Ans.}$$

(47) By formula **3**, $r_2 = \frac{r_1 D^2}{d^2}$, where r_1 is the original resistance of a round conductor, r_2 is the resistance after its diameter has been changed, D is its original diameter, and d is its changed diameter. In this example, $r_1 = 86.5$ ohms, $D = .1$ inch, and $d = .02$ inch; hence, $r_2 = \frac{r_1 D^2}{d^2} = \frac{86.5 \times .1^2}{.02^2} = \frac{86.5 \times .01}{.0004} = 2{,}162.5$ ohms. Ans.

(48) By formula **4**, $r_2 = r_1 (1 + t k)$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its temperature has risen, k is the temperature coefficient, and t is the number of degrees rise Fahrenheit. In this example, $r_1 = 91.8$ ohms, $t = 72 - 45 = 27$ degrees, and $k = .000244$, from Table 1. Hence, $r_2 = r_1 (1 + t k) = 91.8\ (1 + 27 \times .000244) = 91.8 \times 1.006588 = 92.4048$ ohms. Ans.

(49) By formula **5**, $r_2 = \frac{r_1}{1 + t k}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its temperature has fallen, t is the number of degrees fall Fahrenheit, and k is the temperature coefficient of the material. In this example, $r_1 = .144$ ohm, $t = 87 - 41 = 46$ degrees Fahrenheit, and $k = .002155$, from Table 1. Hence,

$$r_2 = \frac{r_1}{1 + t k} = \frac{.144}{1 + 46 \times .002155} = \frac{.144}{1.09913} = .131 \text{ ohm.}$$

Ans.

(50) First reduce the specific resistance in microhms to the resistance in ohms by dividing by 1,000,000, Art. **44**, which gives $\frac{3.565}{1,000,000} = .000003565$ ohm; or, in other words, the resistance of a block of platinum one inch long, and whose sectional area is one square inch, is .000003565 ohm. Next, from this resistance and length, calculate the resistance of 126 feet of platinum with a sectional area of 1 sq. in., by using formula **1**, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its length has been changed, l_1 is the original length of the conductor, and l_2 is its changed length. In this example, $r_1 = .000003565$ ohm, $l_1 = 1$ inch, and $l_2 = 126 \times 12 = 1,512$ inches. Hence,

$$r_2 = \frac{r_1 l_2}{l_1} = \frac{.000003565 \times 1,512}{1} = .00539028 \text{ ohm};$$

that is, the resistance of 126 feet of platinum having a sectional area of 1 sq. in. is .00539 ohm, nearly. From this result calculate the resistance of 126 feet of platinum when its sectional area $= .1^2 \times .7854 = .007854$ sq. in., by using formula **2**, $r_2 = \frac{r_1 a_1}{a_2}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its sectional area is changed, a_1 is its original sectional area, and a_2 is its changed sectional area. At this stage of the example, $r_1 = .00539$ ohm, $a_1 = 1$ sq. in., $a_2 = .007854$ sq. in. Hence, $r_2 = \frac{.00539 \times 1}{.007854} = .686$ ohm. Ans.

(51) From Art. **53**, the fundamental equation of the Wheatstone bridge is $X = \frac{M}{N} \times P$, where X is the unknown resistance, M is the resistance of the upper balance arm, N is the resistance of the lower balance arm, and P is the resistance of the adjustable arm. It will be seen from the connections of the battery and galvanometer circuits in the diagram that the coils lying between c and a form the upper balance arm of the bridge, and hence, in

this example, $M = 1$ ohm; the coils between a and d form the lower balance arm, and hence $N = 100$ ohms; the coils between d and b form the adjustable arm, and hence $P = 500 + 200 + 20 + 2 + 1 = 723$ ohms. Substituting these values in the fundamental equation gives

$$X = \frac{M}{N} \times P = \frac{1}{100} \times 723 = 7.23 \text{ ohms. Ans.}$$

(52) By formula **6**, $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total E. M. F. in volts developed in the circuit, and R is the total resistance in ohms of the circuit. In this example, $E = 36$ volts and $R = 24 + 18 = 42$ ohms; since, Art. **60**, the total resistance of a closed circuit is the sum of the internal and external resistances. Hence, $C = \frac{E}{R} = \frac{36}{42} = .8571$ ampere. Ans.

(53) By formula **7**, $R = \frac{E}{C}$, where R is the total resistance in ohms of a closed circuit, E is the total E. M. F. in volts developed in the circuit, and C is the current in amperes flowing through the circuit. In this example, $E = 12.6$ volts and $C = 2.7$ amperes; hence, $R = \frac{E}{C} = \frac{12.6}{2.7} = 4.6667$ ohms. Ans.

(54) By formula **8**, $E = C R$, where E is the total E. M. F. in volts developed in a closed circuit, C is the current in amperes flowing through the circuit, and R is the total resistance of the circuit. In this example, $C = .8$ ampere and $R = 31.5 + 11 = 42.5$ ohms, since, Art. **60**, the total resistance of a closed circuit is the sum of the internal and external resistances. Therefore, $E = C R = .8 \times 42.5 = 34$ volts. Ans.

(55) By Art. **64** and formula **8**, $E' = C R'$, where E' is the difference of potential in volts between two points in a circuit, C the current in amperes flowing through the circuit, and R' the resistance of the circuit between the

two points. In this example, $C = .12$ ampere and $R' = 204$ ohms; hence, $E' = C R' = .12 \times 204 = 24.48$ volts. Ans.

(56) (*a*) By Art. **64** and formula **7**, $R' = \frac{E'}{C}$, where R' is the total resistance in ohms between two points in a circuit, E' the drop, or loss, of potential in volts between the two points, and C the current in amperes flowing in the circuit. In this example, the two conductors leading to and from the receptive device can be considered as in series, forming one single conductor 1,200 feet in length, in which the drop, or loss, of potential is 10% of 250 volts, or $.10 \times 250 = 25$ volts; that is, $E' = 25$ volts. Since $C = 80$ amperes, then $R' = \frac{E'}{C} = \frac{25}{80} = .3125$ ohm; or, in other words, the sum of the resistances of two conductors which transmit a current of 80 amperes to and from the receptive device with a loss of 25 volts is .3125 ohm. Ans.

(*b*) The resistance per foot of any conductor is found by dividing its total resistance by its length in feet. Assume the two conductors leading to and from the receptive device to be one single conductor 1,200 feet in length and offering a resistance of .3125 ohm; hence, its resistance per foot is

$$\frac{.3125}{1,200} = .00026 \text{ ohm. Ans.}$$

(57) By formula **6**, $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total E. M. F. in volts developed in the circuit, and R is the total resistance in ohms of the circuit. In this example, $E = 24$ volts and $R = 8.1 + 15.9 = 24$ ohms, since, Art. **60**, the total resistance of a closed circuit is the sum of the internal and external resistances. Hence,

$$C = \frac{E}{R} = \frac{24}{24} = 1 \text{ ampere.}$$

By formula **9**, $E' = E - C r_i$, where E' is the available, or external, E. M. F. in volts of a battery or other electric source in a closed circuit, E is the total E. M. F. in volts

developed in the source, C is the current in amperes flowing through the circuit, and r_i is the internal resistance of the battery or electric source. In this example, $E = 24$ volts, $C = 1$ ampere, and $r_i = 8.1$ ohms. Hence, $E' = E - Cr_i = 24 - (8.1 \times 1) = 15.9$ volts. Ans.

(58) Let A represent the first branch and B the second; then, $r_1 = 1.2$ ohms, $r_2 = 2.2$ ohms, and $C = 45$ amperes.

The current c_1 in branch A will then be found by substituting these values in formula **10**, which gives

$$c_1 = \frac{C r_2}{r_1 + r_2} = \frac{45 \times 2.2}{1.2 + 2.2} = \frac{99}{3.4} = 29.1176 \text{ amperes. Ans.}$$

Since the sum of the currents in the two branches is 45 amperes, the current in branch B is, therefore, the difference between 45 amperes and the current in branch A, or $45 - 29.1176 = 15.8824$ amperes. Ans.

(59) By formula **12**, the joint resistance of two conductors connected in parallel is equal to the product of their separate resistances divided by their sum, or $\frac{r_1 r_2}{r_1 + r_2}$, where r_1 and r_2 are the separate resistances of the two branches. In this example, $r_1 = 45$ ohms and $r_2 = 63$ ohms. Hence, $\frac{45 \times 63}{45 + 63} = 26.25$ ohms, the joint resistance of the two conductors connected in parallel.

(60) From Art. **72**, the joint resistance of three conductors connected in parallel is given by formula **13**, $R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}$, where r_1, r_2, and r_3 are the separate resistances of the three conductors. In this example, let $r_1 = 414$ ohms, $r_2 = 810$ ohms, and $r_3 = 1{,}206$ ohms. Then, the joint resistance of the three conductors A, B, and C when connected in parallel is

$$\frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} = \frac{414 \times 810 \times 1{,}206}{810 \times 1{,}206 + 414 \times 1{,}206 + 414 \times 810} =$$

$$\frac{404{,}420{,}040}{976{,}860 + 499{,}284 + 335{,}340} = \frac{404{,}420{,}040}{1{,}811{,}484} = 223.2534 \text{ ohms.}$$

Ans.

DYNAMOS AND MOTORS.

(PART 2.)

(1) By formula **1**, $E = \frac{2NSn}{10^8}$. In this example, $N = 6{,}250{,}000$ lines of force, $S = 100$ outside, or face, wires in series, for if 200 turns were wound around the core, there would be 200 outside, or face, wires, and, from Art. **23**, one-half would be connected in series, and $n = \frac{1200}{60}$ revolutions per second. Substituting these values in above formula gives $E = \frac{2NSn}{10^8} = \frac{2 \times 6{,}250{,}000 \times 100 \times 1{,}200}{100{,}000{,}000 \times 60} =$ 250 volts. Ans.

(2) From Art. **36**, it will be seen that the current in the shunt field of a dynamo is equal to the difference of potential between the brushes divided by the resistance of the shunt-field circuit, or $C_s = \frac{E_t}{R_s}$. In this example, $E_t =$ 220 volts and $R_s = 440$ ohms; hence, $C_s = \frac{E_t}{R_s} = \frac{220}{440} = .5$ ampere. Ans.

(3) See Arts. **13** and **14**.

(4) In Art. **1**, it is stated that a current will be induced in a closed coil or circuit when there is a change in the number of lines of force passing through that coil or circuit. In this case, as the magnetic field is uniform, there is no change in the number of lines of force passing through the coil C when it is moved from its original position to the position C', as shown by the dotted outlines; and, hence, no current will flow around the ring.

(5) In order to determine the result in watts, it is necessary to reduce all quantities to the same units; hence, the first operation is to reduce the input from horsepower to watts. From Art. **81**, Part 1, one horsepower is equivalent to 746 watts; therefore, the input $= 18 \times 746 = 13.428$ watts. Then, by formula **5**, the output

$$O = \frac{13,428 \times 88}{100} = 11,816.64 \text{ watts. Ans.}$$

(6) In this example, it is first necessary to determine the input in watts. By formula **4**, the input $I = \frac{100 \times 17,500}{87.5} = 20,000$ watts. According to Art. **64**, it will be seen that the watts lost in the field coils are equal to the input in watts multiplied by the per cent. loss and divided by 100. Hence, the loss $= \frac{20,000 \times 2.6}{100} = 520$ watts. Ans.

(7) In Art. **67**, $C_s = \frac{E_s}{r_s}$. In this example, $E_s = \frac{110}{55} = 2$ amperes. By formula **19**, Part 1, the watts lost in the shunt circuit are equal to the difference of potential between the terminals of that circuit multiplied by the current in amperes flowing through the circuit; or, $W = E \times C$. Substituting, we have $W = 110 \times 2 = 220$ watts. Ans.

(8) See Arts. **14** and **19.**

(9) From Ohm's law, the resistance of the field circuit is equal to the difference of potential between the brushes divided by the current in the field circuit. Let E_e be the difference of potential between the brushes of the dynamo, C_1 be the strength of current when the resistance is all in circuit, and C_2 be the strength of current when the resistance is cut out, or short-circuited. If r_1 represents the resistance of the field circuit, including that of the rheostat, then $r_1 = \frac{E_e}{C_1} = \frac{360}{1.5} = 240$ ohms; and if r_2 represents the resistance

of the field circuit when the resistance of the rheostat has been cut out, or short-circuited, then $r_s = \frac{E_s}{C_s} = \frac{360}{1.8} =$ 200 ohms. Hence, the amount of resistance which was cut out, or short-circuited, in the rheostat is the difference between these two resistances, or $240 - 200 = 40$ ohms. Ans.

(10) Use formula **4.** In this example, the input $= \frac{100 \times 65,000}{90.5} = 71,823.2044$ watts. Since one horsepower equals 746 watts, then 71,823.2044 watts equal $\frac{71,823.2044}{746} =$ 96.2778 horsepower. Ans.

(11) If the current circulates in the direction indicated by the arrow-heads, *neither* pole-piece will be a north pole; for, by applying the rule given in Art. **29,** Part 1, it will be seen that the north pole of one field coil is opposite the south pole of the other, and the lines of force circulate around the magnets without passing through the armature. If the winding of the right-hand coil were reversed, its top would be a north pole, and the top of the left-hand coil being also north, the pole-piece P would become a north consequent pole.

(12) See Arts. **19** and **21.**

(13) In this example, it is first necessary to change the input from horsepower to watts. Since one horsepower is equivalent to 746 watts, then 10 horsepower is equivalent to $10 \times 746 = 7,460$ watts. Then, by formula **2,** the efficiency $\text{E} = \frac{6,341 \times 100}{7,460} = 85$ per cent. Ans.

(14) From b to a through the conductor; for, by applying the thumb-and-finger rule given in Art. **8,** it will be seen that the middle finger points towards a from b.

(15) In this example, it is first necessary to find the input in watts. The output $= 11,900$ watts and the per

cent. efficiency = 85. Then, by formula **4**, the input = $\frac{100 \times 11,900}{85} = 14,000$ watts. According to Art. **64**, the watts lost are found by multiplying the input by the per cent. loss and dividing by 100. Hence, $\frac{14,000 \times 1.8}{100} = 252$ watts lost in core by eddy currents and hysteresis. Ans.

(16) (*a*) See Art. **28.**
(*b*) See Art. **25.**

(17) According to Art. **70** and formula **20**, Part 1, $W_i = C^2 r_i$. In this example $C = 120$ amperes and $r_i = .040$ ohm; hence, $W_i = 120^2 \times .040 = 576$ watts. Ans.

(18) From example 17, the normal output from the dynamo is 120 amperes at 125 volts, or $120 \times 125 = 15,000$ watts. The next step is to determine the input at this output when the efficiency is 75%. By formula **4**, the input in this case is $\frac{100 \times 15,000}{75} = 20,000$ watts. From example 17, there are 576 watts lost in the armature due to its resistance, and, from Art. **72**, the loss in the armature due to its resistance is $\frac{576 \times 100}{20,000} = 2.88\%$. Ans.

(19) (*a*) and (*b*) See Art. **77.**

(20) Under "Field Losses," in Art. **66**, the watts lost in the series coils are found by using formula **20**, Part 1, where $W = C^2 R$. In this example, $C = 40$ amperes and $R = .04$ ohm; hence, $W = 40^2 \times .04 = 40 \times 40 \times .04 = 64$ watts, which represents the loss in the series coils. The watts lost in the shunt coil are given by formula **21**, Part 1, where $W = \frac{E^2}{R}$ In this case, $E = 550$ volts and $R = 550$ ohms; hence, $W = \frac{E^2}{R} = \frac{550 \times 550}{550} = 550$ watts, which is the loss in the shunt field. The total loss in the fields of a compound dynamo is

equal to the sum of the losses in the series and shunt coils. Hence, the total loss in this case is $64 + 550 = 614$ watts. Ans.

(21) P' is the south consequent pole of the field, since, from the rule in Art. **29,** Part 1, in looking through the coils c and d from a position near P', the current is circulating around the field cores in the same direction as the movements of the hands of a watch; while, on the contrary, in looking through the coils a and b from a position near P, the current is circulating around the upper field cores in a direction opposite to the movements of the hands of a watch.

(22) See Art. **29.**

(23) From Art. **64,** the total loss in a dynamo is the sum of the separate losses; hence, in this example, the total loss in watts is $356 + 178 + 263 + 423 + 50 = 1{,}270$ watts. From Art. **59,** the input to the dynamo in this case is $15{,}000 + 1{,}270 = 16{,}270$ watts. By formula **2,** the efficiency $\mathsf{E} = \frac{15{,}000 \times 100}{16{,}270} = 92.1942\%$ at this output. Ans.

(24) (*a*) From example 23, the loss in mechanical friction is 356 watts, and the input is 16,270 watts; hence (see Art. **72**), the per cent. loss is $\frac{356 \times 100}{16{,}270} = 2.1881\%$. Ans.

(*b*) From example 23, the loss in the core by eddy currents and hysteresis is 178 watts, and the input is 16,270 watts; hence (see Art. **72**), the per cent. loss is

$$\frac{178 \times 100}{16{,}270} = 1.094\%. \quad \text{Ans.}$$

(*c*) From example 23, the loss in the field coils is 263 watts, and the input is 16,270 watts; hence, the per cent. loss is $\frac{263 \times 100}{16{,}270} = 1.6165\%$. Ans.

(*d*) From example 23, the loss in the armature ($C^2 r$) = 423 watts, and the input is 16,270 watts; hence, the per cent. loss is $\frac{423 \times 100}{16{,}270} = 2.5999\%$. Ans.

(*e*) **From example 23, the sum of the separate losses is 1,270 watts, and this is the difference between the input and the output; the input is 16,270 watts.** Hence, by formula **3,** the total per cent. loss $L = \frac{100 \times 1,270}{16,270} = 7.8058\%$. Ans.

(25) From Art. **22,** it will be seen that the electromotive force generated in an armature is proportional to the speed, other conditions and quantities remaining unchanged. Hence, in this example, if E represents the electromotive force which is generated when the armature is driven at 1,400 revolutions per minute, then, by proportion, $440 : E = 1,200 : 1,400$, or $E \times 1,200 = 440 \times 1,400$; therefore, $E = \frac{440 \times 1,400}{1,200} = 513\frac{1}{3}$ volts. Ans.

(26) See Art. **73** and those following.

(27) In Art. **6,** under "Mutual Induction," it is stated that when the current in the primary circuit tends to increase in strength, the induced current in the secondary coil will tend to circulate around the core which forms the magnetic circuit of both coils, in the opposite direction to that of the current in the primary circuit. In this case, the current in the primary circuit flows from the positive terminal n of the battery when the circuit is closed, around the coil to the negative terminal m; or, in other words, the current circulates around the core in an opposite direction to the movements of the hands of a watch, as viewed by a person looking through the coil from a position near C. Consequently, the momentary current induced in the secondary coil S would tend to circulate around the core in the reverse direction, that is, in the same direction as the movements of the hands of a watch. The direction of the current in the secondary circuit would, therefore, be from the terminal x through the coil to y, and then through the resistance R to x again.

(28) In Art. **6,** under "Mutual Induction," it is stated that when the strength of the current in the primary circuit

suddenly decreases, the momentary current induced in the secondary coil will circulate around the core which forms the magnetic circuit of both coils in the same direction to that of the current in the primary coil. In this case, when the strength of the current in the primary circuit suddenly decreases, it continues to flow in the same direction as in example 27, that is, from the terminal n through the primary coil P to m, and completing the circuit to n through the battery B. Consequently, the current in the secondary coil S circulates around the core in the same direction, that is, from y through the secondary coil S to x, completing the circuit to y again through the resistance R.

(29) See Arts. **78** and **79.**

(30) Yes; because (Art. **1**) a change takes place in the number of lines which pass through the coil. From the rule given in Art. **7,** it will be seen that the current will circulate around the ring in the same direction as the movements of the hands of a watch; for the effect of the motion is to diminish the number of lines of force which pass through the coil, and the observer is looking along the magnetic field in the direction of the lines of force.

(31) See Art. **42.**

(32) From Art. **11,** it will be seen that the *rate of cutting* lines of force is found by dividing the number cut by the time required to cut them; hence, in this case, the rate of cutting is $\frac{8,000,000}{.25} = 32,000,000$ lines of force per second.

(33) Because the solid iron core would act as a large conductor cutting lines of force at an angle, and thereby producing *local*, or *eddy*, currents in the core, heating it badly, and uselessly dissipating a large amount of energy. (Art. **16.**)

(34) By formula **1,** $E = \frac{2 N S n}{10^8}$. In this example, if 150 complete turns of wire are wound upon the drum core,

there will be 300 outside, or face, wires, and one-half of these will be connected in series, as explained in Art. **21**; then, $S = 150$ outside wires connected in series, $N = 2{,}500{,}000$ lines of force, and $n = \frac{1020}{60}$. Hence,

$$E = \frac{2NSn}{10^8} = \frac{2 \times 2{,}500{,}000 \times 150 \times 1{,}020}{100{,}000{,}000 \times 60} = 127.5 \text{ volts. Ans.}$$

(35) From Art. **22**, it will be seen that the electromotive force generated in an armature is proportional to the number of lines of force passing through the core. Let E represent the electromotive force which is generated when 1,250,000 lines of force are passing through the core; then, by proportion, $200 : E = 750{,}000 : 1{,}250{,}000$, or $E \times 750{,}000 = 200 \times 1{,}250{,}000$; therefore, $E = \frac{200 \times 1{,}250{,}000}{750{,}000} = 333\frac{1}{3}$ volts. Ans.

(36) (*a*) See Art. **65.**
(*b*) See Art. **70.**
(*c*) See Art. **66.**

(37) Towards the side *a*; for by applying the thumb-and-finger rule given in Art. **26**, and making the forefinger point in the direction of the lines of force and the middle finger in the direction of the current, the thumb will point towards the side *a*.

(38) Use the formula given under "Field Losses" in Art. **67**, $C_s = \frac{E_e}{r_s}$, which is a modification of formula **6**, Part 1. In this example, $E_e = 525$ volts and $r_s = 650$ ohms; hence, $C_s = \frac{E_e}{r_s} = \frac{525}{650} = .8076$ ampere. Ans.

(39) The increase in voltage from no load to full load is $124.2 - 115 = 9.2$ volts, which is $\frac{9.2 \times 100}{115} = 8\%$ of the normal voltage. Therefore, the over-compounding is 8%. Ans.

(40) See Art. **20.**

(41) First change the input from horsepower to watts. Since 1 horsepower is equivalent to 746 watts, 44 horsepower is equivalent to $44 \times 746 = 32{,}824$ watts. By formula **2,** the efficiency

$$E = \frac{100 \times 29{,}820}{32{,}824} = 90.8481\%. \quad \text{Ans.}$$

(42) In this example, the input $I = 20{,}000$ watts and the output $O = 17{,}500$ watts. Then, by formula **3,** the per cent. loss $L = \dfrac{100 \times (20{,}000 - 17{,}500)}{20{,}000} = 12.5\%$. Ans.

(43) In this example, the output is 12,500 watts and the efficiency is 92.5%. Consequently, by formula **4,** the input $I = \dfrac{100 \times 12{,}500}{92.5} = 13{,}513.5135$ watts. Reducing this input from watts to horsepower gives

$$\frac{13{,}513.5135}{746} = 18.1146 \text{ horsepower.} \quad \text{Ans.}$$

(44) See Art. **73.**

(45) In this example, it is first necessary to change the input from horsepower to watts. Since one horsepower is equivalent to 746 watts, fifty-five horsepower is equivalent to $55 \times 746 = 41{,}030$ watts. The output of the dynamo, by formula **5,** $= \dfrac{41{,}030 \times 88.5}{100} = 36{,}311.55$ watts. Ans.

(46) In the same manner as shown in Art. **64,** it will be seen that the loss in watts in the field coils is equal to the input multiplied by the per cent. loss and divided by 100. In this example, it is first necessary to change the input from horsepower to watts. Since one horsepower is equivalent to 746 watts, forty-five horsepower is equivalent to $45 \times 746 = 33{,}570$ watts. Consequently, the loss in the field coils is $\dfrac{33{,}570 \times 2}{100} = 671.4$ watts. Ans.

(47) See Arts. **34, 36,** and **39.**

(48) From Art. **72,** the per cent. loss in the core is found by dividing the watts lost in the core by the input and multiplying by 100. Reducing 64 horsepower to watts gives $64 \times 746 = 47{,}744$ watts. Consequently, the loss in the core is $\frac{800 \times 100}{47{,}744} = 1.6756\%$. Ans.

(49) See Art. **76.**

(50) See Art. **43.**

DYNAMOS AND MOTORS.

(PART 3.)

(1) (*a*) and (*b*) See Art. **40.**

(2) See Art. **54.**

(3) See Art. **94.**

(4) Drum. See Art. **27.**

(5) See Art. **1.**

(6) See Art. **37.**

(7) See Art. **31.**

(8) An open circuit in the armature winding, probably in the lead to the burned commutator segment. See Art. **97.**

(9) (*a*) See Art. **67.**

(*b*) See Art. **56.**

(10) (*a*) and (*b*) See Art. **24.**

(11) See Art. **21.**

(12) See Art. **36.**

(13) (*a*) The length of the arm of the brake being 36 inches, or 3 feet, the torque of the motor is $27 \times 3 = 81$ foot-pounds (Arts. **62** and **63**). The revolutions per minute being 900, the H. P. output of the motor is, from formula **3**,

$$\text{H. P.} = \frac{2 \times 3.1416\, TS}{33,000} = \frac{6.2832 \times 81 \times 900}{33,000} = 13.88 \text{ H. P. Ans.}$$

(*b*) To find the efficiency, it is first necessary to find the input and reduce the input and the output to the same units (Art. **63**). In this case, the input is $25 \times 480 =$ 12,000 watts. Reducing 13.88 H. P. to watts, $13.88 \times 746 =$ 10,354 watts. Then, by formula **2,** Part 2, the efficiency

$$E = \frac{100 \times 10,354}{12,000} = 86.3\%. \quad \text{Ans.}$$

(14) See Art. **68.**

(15) (*a*) and (*b*) See Art. **99.**

(16) The connections would be about as shown in Fig. 1, in which *F* is the field circuit, *B*, B_1 the brushes of the motor, *R* the starting resistance, *R. S.* the reversing-switch, *F. B.* the fuse boxes, and *S* the main switch.

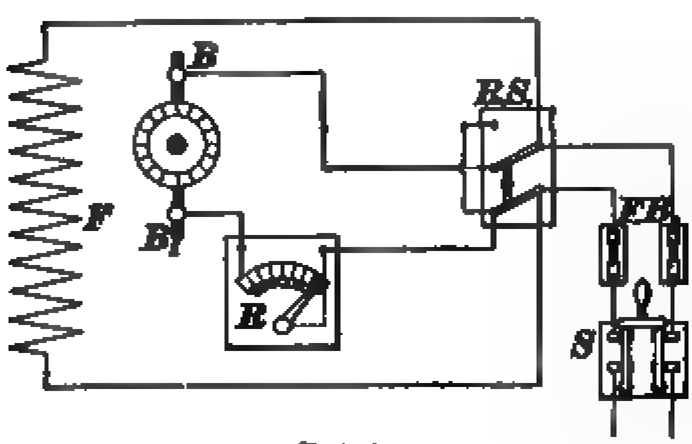


FIG. 1.

(17) By varying the applied E. M. F. or the strength of the field. Art. **57.**

(18) See Art. **39.**

(19) Because the self-induction of the coil having the lesser E. M. F. prevents the flow of current. Art. **14.**

(20) Circuit No. *4* (Fig. 48, Art. **115**) would first be short-circuited by plugging in a cable from terminal −*3* to terminal −*4*. The cable from terminal −*3* to terminal +*4* may now be removed from terminal +*4* and connected to terminal +*1*, and a cable plugged in from terminal −*1* to terminal −*B*. Then, by pulling out the cable from terminal −*3* to terminal −*4*, circuit No. *1* is connected in series with circuit No. *3*, on dynamo *B*, as required. The cable from terminal −*4* to terminal −*B* should be removed to make circuit No. *4* "dead."

(21) (*a*) See Art. **103.**

(*b*) See Arts. **92** and **93.**

(22) (*a*) See Art. **3.**

(*b*) See Art. **5.**

(23) (*a*) 29 amperes flowing out.

(*b*) See Art. **35.**

(24) See Art. **69.**

(25) The input to the motor being $33 \times 230 = 7{,}590$ watts, and the efficiency being 85%, the output, by formula **5,** Part 2, is $\frac{7{,}590 \times 85}{100} = 6{,}451.5$ watts. This is equal to $\frac{6{,}451.5}{746} = 8.65$ H. P. The arm of the brake being 2 feet long and the pressure on the scale platform being 20 lb., the torque of the motor must be 40 foot-pounds $= T$. Knowing the H. P. and the torque, the speed may be found from formula **4,** $S = \frac{33{,}000 \text{ H. P.}}{2 \times 3.1416\, T}$. Substituting the above values for H. P. and T,

$$S = \frac{33{,}000 \times 8.65}{2 \times 3.1416 \times 40} = \frac{285{,}450}{251.328} = 1{,}136 \text{ rev. per min.} \quad \text{Ans.}$$

(26) The frequency is equal to the number of revolutions per second multiplied by the number of pairs of poles (Art. **26**). In this case, $\frac{1080}{60} \times \frac{14}{2} = 126$ cycles per second. Ans.

(27) (*a*) and (*b*) See Art. **118.**

(28) See Art. **58.**

(29) See Art. **78.**

(30) See Art. **101.**

(31) (*a*) See Art. **32.**

(*b*) See Art. **31.**

(32) See Arts. **29** and **30.**

(33) See Art. **85.**

(34) See Arts. **114** and **132.**

(35) See Art. **17** and Fig. 6.

(36) See Art. **50.**

(37) See Art. **124.**

(38) See Art. **80.**

(39) See Art. **12.**

(40) See Art. **127.**

(41) See Art. **90.**

(42) See Art. **55.**

(43) See Art. **106.**

(44) See Art. **109.**

(45) See Art. **65.**

(46) See Art. **128.**

(47) See Art. **23.**

(48) See Art. **41.**

(49) See Art. **131.**

(50) When in the position of least action, a coil is momentarily disconnected from the external circuit, then is thrown in parallel with the coil ahead of it, then in series with the other two coils which are then in parallel, then in parallel with the coil behind it, and then disconnected from the circuit again. See Art **20,** also Fig. 7.

(51) (*a*) See Art. **68.**
(*b*) See Arts **80** and **84.**

(52) (*a*) Of the 5 amperes input, by Ohm's law, $\frac{125}{62.5} =$ 2 amperes go to the field, the loss being, therefore, $2 \times 125 = 250$ watts The rest, or $3 \times 125 = 375$ watts, make up the friction and core losses of the machine (Art. **65**).

When taking an input of 77 amperes at 125 volts, or 9,625 watts, there would still be required 250 watts for the field and 375 watts for the core losses and friction. Of the 77 amperes, 75 flow through the armature, and as this has a resistance of .04 ohm, the armature $C^2 r$ would be $75^2 \times .04 = 225$ watts. The total losses would then be $250 + 375 + 225 = 850$ watts, and the output would, therefore, be $9,625 - 850 = 8,775$ watts, or $\frac{8775}{746} = 11.76$ H. P. Ans.

(*b*) The output being 8,775 watts and the input 9,625, the efficiency is, by formula **2**, Part 2, $\frac{100 \times 8,775}{9,625} = 91.17$ per cent. Ans.

(53) See Art. **104.**

(54) (*a*) and (*b*) See Art. **26.**

(55) See Art. **10.**

(56) See Art. **54.**

(57) (*a*) and (*b*) See Art. **121** and Figs. 51 and 52.

(58) From Art. **79,** the speed of the field would be $\frac{72}{5} =$ 14.4 revolutions per second, or $14.4 \times 60 = 864$ revolutions per minute. With 2.5% slip, the speed of the armature would be $864 - (864 \times .025) = 842.4$ revolutions per minute. Ans.

(59) See Art. **92.**

(60) (*a*) See Art. **52.**

(*b*) and (*c*) See Art. **53.**

(61) When the whole of the coil is directly under one pole-piece. See Art. **111.**

(62) See Art. **73.**

(63) See Art. **26.**

(64) See Art. **130.**

(65) See Art. **59.**

(66) See Art. **94.**

H. M. IV.—14

(67) (*a*) and (*b*) See Art. **56.**
(*c*) See Arts. **55** and **56.**

(68) See Arts. **21** and **22.**

(69) See Art. **96.**

(70) See Art. **79.**

(71) See Art. **52.**

(72) See Art. **41.**

(73) See Art. **79.**

(74) There is no definite answer for this problem, as a great number of different arrangements is possible. See Art. **129.** By comparing it with Fig. 54, Art. **127,** it will be seen, if the connections are correctly made and all the necessary instruments in place, the exact arrangement is a matter of taste and judgment.

(75) See Art. **63.**

(76) See Art. **27.**

(77) The frequency being 132 and there being 11 pairs of poles, the motor will run at $\frac{132}{11} = 12$ revolutions per second, or $60 \times 12 = 720$ revolutions per minute. See Art. **68.**

(78) (*a*) and (*b*) See Art. **80.**

(79) See Art. **6.**

(80) See Art. **57.**

(81) See Art. **97.**

(82) See Art. **6.**

(83) The effective voltage of the alternating current is obtained by using formula **2,** Art. **47,**

$$\bar{E} = .612 \times 200 = 122.4 \text{ volts.} \quad \text{Ans.}$$

(84) (*a*) The strength of the direct current would be the same as the *effective* strength of the alternating current.

which is .707 of its maximum strength. See Art. **36.** Then, $.707 \times 12 = 8.48$ amperes. Ans.

(*b*) See Art. **36.**

(85) The counter E. M. F. E' depends on the field strength and on the speed of rotation of the armature. Hence, if the field is weakened, E' will decrease and allow a larger current to flow through the armature. This will result in an increase in speed, till at the new speed and with the weakened field, the counter E. M. F. is raised sufficiently to cut down the current to its proper value, or so that the torque will again balance the resistance to rotation. See Art. **57.**

(86) See Art. **83.**

(87) See Art. **71.**

(88) See Art. **81.**

(89) See Art. **83.**

(90) See Art. **46.** The E. M. F. of each phase will be given by formula **1**,

$$\bar{E} = .707 \times 220 = 155.5 \text{ volts. Ans.}$$

(91) See Art. **48.**

(92) See Art. **34.**

A TEXTBOOK

ON

MARINE ENGINEERING

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TABLES AND FORMULAS

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B–2

TABLES AND FORMULAS.

This volume contains all the principal Tables, Rules, and Formulas occurring in the Instruction Papers of the Course. They have been collected and placed in this volume in order to make them convenient for ready reference, so that the student will not be obliged to search the Instruction Papers to find them.

The various Rules and Formulas are here grouped under the titles of the Instruction Papers in which they occur. Following each rule and formula are its number and also that of the article of the Instruction Paper in which it is discussed. Although these numbers do not run consecutively, they may be readily found in the text by noting to what Instruction Paper they belong. Thus: on page 26 of this volume, we see that the formula for "Horsepower of Gears" is followed by "Art. **557.**" Turning back the pages, we find that this formula occurs under the heading "Formulas Used in Mechanics." Therefore, in Art. **557** of the Instruction Paper *Mechanics*, we shall find the discussion of the formula referred to.

TABLES.

SPECIFIC GRAVITIES AND WEIGHTS PER CUBIC FOOT.

METALS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Osmium	23.00	1,437.5
Platinum	21.50	1,343.8
Gold	19.50	1,218.8
Mercury	13.60	850.0
Lead (cast)	11.35	709.4
Silver	10.50	656.3
Copper (cast)	8.79	549.4
Brass	8.38	523.8
Wrought Iron	7.68	480.0
Cast Iron	7.21	450.0
Steel	7.84	490.0
Tin (cast)	7.29	455.6
Zinc (cast)	6.86	428.8
Antimony	6.71	419.4
Aluminum	2.50	156.3

WOODS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Ash	.845	52.80
Beech	.852	53.25
Cedar	.561	35.06
Cork	.240	15.00
Ebony (American)	1.331	83.19
Lignum-vitæ	1.333	83.30
Maple	.750	46.88
Oak (old)	1.170	73.10
Spruce	.500	31.25
Pine (yellow)	.660	41.20
Pine (white)	.554	34.60
Walnut	.671	41.90

LIQUIDS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Acetic Acid	1.062	66.4
Nitric Acid	1.217	76.1
Sulphuric Acid	1.841	115.1
Muriatic Acid	1.200	75.0
Alcohol	.800	50.0
Turpentine	.870	54.4
Sea Water (ordinary)	1.026	64.1
Milk	1.032	64.5

GASES.

At 32° F., and under a Pressure of One Atmosphere.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Atmospheric Air	1.0000	.08073
Carbonic Acid	1.5290	.12344
Carbonic Oxide	.9674	.07810
Chlorine	2.4400	.19700
Oxygen	1.1056	.08925
Nitrogen	.9736	.07860
Smoke (bituminous coal)	.1020	.00815
Smoke (wood)	.0900	.00727
*Steam at 212° F.	.4700	.03790
Hydrogen	.0692	.00559

* The specific gravity of steam at any temperature and pressure compared with air at the same temperature and pressure is 0.622.

MISCELLANEOUS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Emery	4.00	250
Glass (average)	2.80	175
Chalk	2.78	174
Granite	2.65	166
Marble	2.70	169
Stone (common)	2.52	158
Salt (common)	2.13	133
Soil (common)	1.98	124
Clay	1.93	121
Brick	1.90	118
Plaster Paris (average)	2.00	125
Sand	1.80	113

COEFFICIENTS OF FRICTION.

Description of Surfaces in Contact.	Disposition of Fibers.	State of the Surfaces.	Coefficient of Friction.
Oak on oak	Parallel	Dry	.48
Oak on oak	Parallel	Soaped	.16
Wrought Iron on oak	Parallel	Dry	.62
Wrought Iron on oak	Parallel	Soaped	.21
Cast Iron on oak	Parallel	Dry	.49
Cast Iron on oak	Parallel	Soaped	.19
Wrought Iron on cast iron	—	Slightly Unctuous	.18
Wrought Iron on bronze	—	Slightly Unctuous	.18
Cast Iron on cast iron	—	Slightly Unctuous	.15

TENSILE STRENGTHS OF MATERIALS.

Materials.	Breaking Stress in Pounds per Square Inch.	Working Stress in Pounds per Square Inch.
Timber...........	10,000	600 to 1,200
Cast Iron.........	16,000	1,500 to 3,500
Wrought Iron.....	50,000	5,000 to 12,000
Steel..............	70,000	6,000 to 13,000

CRUSHING STRENGTHS OF MATERIALS.

Materials.	Crushing Strength in Tons per Square Inch.
Cast Iron....................	40
Wrought Iron................	18
Mild Steel	26
Cast Copper..................	5
Cast Brass...................	4.5
Timber (dry)	3.5
Brick	1
Stone	3

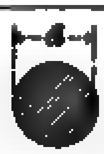
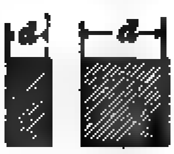
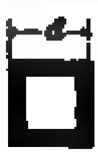
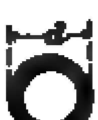
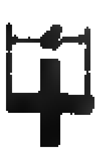
SHEARING STRENGTHS OF MATERIALS.

Materials.	Greatest Shearing Stress in Pounds per Square Inch.	Safe Shearing Stress in Pounds per Square Inch.
Cast Iron.........	18,000	1,500 to 3,000
Wrought Iron.....	40,000	4,000 to 10,000
Steel	60,000	5,000 to 12,000

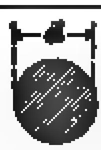
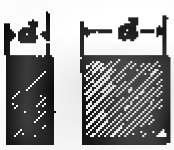
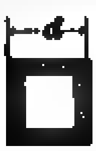
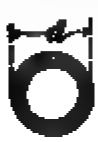
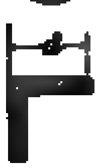
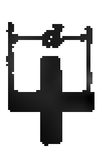
CONSTANTS FOR LINE SHAFTING.

Material of Shaft.	No Pulleys Between Bearings.	Pulleys Between Bearings.
Steel or Cold-Rolled Iron	65	85
Wrought Iron....	70	95
Cast Iron.............	90	120

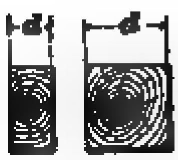
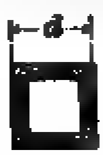
CONSTANTS FOR WROUGHT-IRON PILLARS.

Cross-section of Pillar.		When Both Ends of the Pillar are Flat or Fixed.	When One End of the Pillar is Flat or Fixed, and the Other Round or Movable.	When Both Ends of the Pillar are Round or Movable.
	Round.	2,250	1,500	1,125
	Square or Rectangle.	3,000	2,000	1,500
	Thin Square Tube.	6,000	4,000	3,000
	Thin Round Tube.	4,500	3,000	2,250
	Angle with Equal Sides.	1,500	1,000	750
	Cross with Equal Arms	1,500	1 000	750
	I Beam.	$3,000 \times \frac{A}{A+B}$	$2,000 \times \frac{A}{A+B}$	$1,500 \times \frac{A}{A+B}$

CONSTANTS FOR CAST-IRON PILLARS.

Cross-section of Pillar.	When Both Ends of the Pillar are Flat or Fixed.	When One End of the Pillar is Flat or Fixed, and the Other Round or Movable.	When Both Ends of the Pillar are Round or Movable.
 Round.	281.25	187.5	140.625
 Square or Rectangle.	375.	250.	187.5
 Thin Square Tube.	750.	500.	375.
 Thin Round Tube.	562.5	375.	281.25
 Angle with Equal Sides.	187.5	125.	93.75
 Cross with Equal Arms.	187.5	125.	93.75
 I Beam.	$375 \times \frac{A}{A+B}$	$250 \times \frac{A}{A+B}$	$125 \times \frac{A}{A+B}$

CONSTANTS FOR WOODEN PILLARS.

Cross-section of Pillar.		When Both Ends of the Pillar are Flat or Fixed.	When One End of the Pillar is Flat or Fixed, and the Other Round or Movable.	When Both Ends of the Pillar are Round or Movable.
	Round.	187.5	125.	93.75
	Square or Rectangle.	250.	166.66	125.
	Hollow Square Made of Boards.	500.	333.33	250.

CONSTANTS FOR TRANSVERSE STRENGTH OF BEAMS.

Materials.	Safe Transverse Strength in Pounds.	Materials.	Safe Transverse Strength in Pounds.
Metals:		Woods:	
Cast Iron	100	Birch	35
Wrought Iron...	150	Elm	25
Structural Steel.	160	Ash............	45
Copper	50	Beech	30
Brass	55	Hickory	50
		Maple..........	60
		Oak (American).	45
		Pine (Pitch)	40
		Pine (White). . .	30

THE PROPERTIES OF SATURATED STEAM.

Pressure above Vacuum in Pounds per Square Inch.	Temperature, Fahrenheit Degrees.	Quantities of Heat in British Thermal Units.			Weight of a Cubic Foot of Steam in Pounds.	Volume.	
		Required to Raise Temperature of the Water from 32° to $t°$.	Total Latent Heat at Pressure P.	Total Heat above 32°.		Of a Pound of Steam in Cubic Feet.	Ratio of Vol. of Steam to Vol. of Eq. Weight of Dist. Water at Temp. of Maximum Density.
1	2	3	[illegible]	5	6	7	8
p	t	q	L	H	W	V	R
1	102.018	70.040	1043.015	1113.055	.003027	330.4	20623
2	126.302	94.368	1026.094	1120.462	.005818	171.9	10730
3	141.654	109.764	1015.380	1125.144	.008522	117.3	7325
4	153.122	121.271	1007.370	1128.641	.011172	89.51	5588
5	162.370	130.563	1000.899	1131.462	.013781	72.56	4530
6	170.173	138.401	995.441	1133.842	.016357	61.14	3816
7	176.945	145.213	990.695	1135.908	.018908	52.89	3302
8	182.952	151.255	986.485	1137.740	.021436	46.65	2912
9	188.357	156.699	982.690	1139.389	.023944	41.77	2607
10	193.284	161.660	979.232	1140.892	.026437	37.83	2361
11	197.814	166.225	976.050	1142.275	.028911	34.59	2159
12	202.012	170.457	973.098	1143.555	.031376	31.87	1990
13	205.929	174.402	970.346	1144.748	.033828	29.56	1845
14	209.604	178.112	967.757	1145.869	.036265	27.58	1721
14.69	212.000	180.531	966.069	1146.600	.037928	26.37	1646
15	213.067	181.608	965.318	1146.926	.038688	25.85	1614
16	216.347	184.919	963.007	1147.926	.041109	24.33	1519
17	219.452	188.056	960.818	1148.874	.043519	22.98	1434
18	222.424	191.058	958.721	1149.779	.045920	21.78	1359
19	225.255	193.918	956.725	1150.643	.048312	20.70	1292

1	2	3	4	5	6	7	8
p	*t*	*q*	*L*	*H*	*W*	*V*	*R*
20	227.964	196.655	954.814	1151.469	.050696	19.73	1231.0
22	233.069	201.817	951.209	1153.026	.055446	18.04	1126.0
24	237.803	206.610	947.861	1154.471	.060171	16.62	1038.0
26	242.225	211.089	944.730	1155.819	.064870	15.42	962.3
28	246.376	215.293	941.791	1157.084	.069545	14.38	897.6
30	250.293	219.261	939.019	1158.280	.074201	13.48	841.3
32	254.002	223.021	936.389	1159.410	.078839	12.68	791.8
34	257.523	226.594	933.891	1160.485	.083461	11.98	748.0
36	260.883	230.001	931.508	1161.509	.088067	11.36	708.8
38	264.093	233.261	929.227	1162.488	.092657	10.79	673.7
40	267.168	236.386	927.040	1163.426	.097231	10.28	642.0
42	270.122	239.389	924.940	1164.329	.101794	9.826	613.3
44	272.965	242.275	922.919	1165.194	.106345	9.403	587.0
46	275.704	245.061	920.968	1166.029	.110884	9.018	563.0
48	278.348	247.752	919.084	1166.836	.115411	8.665	540.9
50	280.904	250.355	917.260	1167.615	.119927	8.338	520.5
52	283.381	252.875	915.494	1168.369	.124433	8.037	501.7
54	285.781	255.321	913.781	1169.102	.128928	7.756	484.2
56	288.111	257.695	912.118	1169.813	.133414	7.496	467.9
58	290.374	260.002	910.501	1170.503	.137892	7.252	452.7
60	292.575	262.248	908.928	1171.176	.142362	7.024	438.5
62	294.717	264.433	907.396	1171.829	.146824	6.811	425.2
64	296.805	266.566	905.900	1172.466	.151277	6.610	412.6
66	298.842	268.644	904.443	1173.087	.155721	6.422	400.8
68	300 831	270.674	903.020	1173.694	.160157	6.244	389.8
70	302.774	272.657	901.629	1174.286	.164584	6.076	379.3
72	304.669	274.597	900.269	1174.866	.169003	5.917	369.4
74	306.526	276.493	898.938	1175.431	.173417	5.767	360.0
76	308.344	278.350	897.635	1175.985	.177825	5.624	351.1
78	310.123	280.170	896.359	1176.529	.182229	5.488	342.6
80	311.866	281.952	895.108	1177.060	.186627	5.358	334.5
82	313.576	283.701	893.879	1177.580	.191017	5.235	326.8
84	315.250	285.414	892.677	1178.091	.195401	5.118	319.5
86	316.893	287.096	891.496	1178.592	.199781	5.006	312.5
[illegible]	318.510	288.750	890.335	1179.085	.204155	4.898	305.8

1	2	3	4	5	6	7	8
p	t	q	L	H	W	V	R
90	320.094	290.373	889.196	1179.569	.208525	4.796	299.4
92	321.653	291.970	888.075	1180.045	.212892	4.697	293.2
94	323.183	293.539	886.972	1180.511	.217253	4.603	287.3
96	324.688	295.083	885.887	1180.970	.221604	4.513	281.7
98	326.169	296.601	884.821	1181.422	.225950	4.426	276.3
100	327.625	298.093	883.773	1181.866	.230293	4.342	271.1
105	331.169	301.731	881.214	1182.945	.241139	4.147	258.9
110	334.582	305.242	878.744	1183.986	.251947	3.969	247.8
115	337.874	308.621	876.371	1184.992	.262732	3.806	237.6
120	341.058	311.885	874.076	1185.961	.273500	3.656	228.3
125	344.136	315.051	871.848	1186.899	.284243	3.518	219.6
130	347.121	318.121	869.688	1187.809	.294961	3.390	211.6
135	350.015	321.105	867.590	1188.695	.305659	3.272	204.2
140	352.827	324.003	865.552	1189.555	.316338	3.161	197.3
145	355.562	326.823	863.567	1190.390	.326998	3.058	190.9
150	358.223	329.566	861.634	1191.200	.337643	2.962	184.9
160	363.346	334.850	857.912	1192.762	.358886	2.786	173.9
170	368.226	339.892	854.359	1194.251	.380071	2.631	164.3
180	372.886	344.708	850.963	1195.671	.401201	2.493	155.6
190	377.352	349.329	847.703	1197.032	.422287	2.368	147.8
200	381.636	353.766	844.573	1198.339	.443310	2.256	140.8
210	385.759	358.041	841.556	1199.597	.464295	2.154	134.5
220	389.736	362.168	838.642	1200.810	.485237	2.061	128.7
230	393.575	366.152	835.828	1201.980	.506139	1.976	123.3
240	397.285	370.008	833.103	1203.111	.527003	1.898	118.5
250	400.883	373.750	830.459	1204.209	.547831	1.825	114.0
260	404.370	377.377	827.896	1205.273	.568626	1.759	109.8
270	407.755	380.905	825.401	1206.306	.589390	1.697	105.9
280	411.048	384.337	822.973	1207.310	.610124	1.639	102.3
290	414.250	387.677	820.609	1208.286	.630829	1.585	99.0
300	417.371	390.933	818.305	1209.238	.651506	1.535	95.8

SPECIFIC HEAT OF VARIOUS SUBSTANCES.

Substance.	Specific Heat.	Substance.	Specific Heat.
Water	1.0000	Ice	.5040
Sulphur	.2026	Steam (superheated)	.4805
Iron	.1138	Air	.2375
Copper	.0951	Oxygen	.2175
Silver	.0570	Hydrogen	3.4090
Tin	.0562	Nitrogen	.2438
Mercury	.0333	Carbon Monoxide	.2479
Lead	.0314	Carbon Dioxide	.2170

RELATIVE POSITIONS OF CRANK AND ECCENTRIC.

	Kind of Valve.	Kind of Rocker-Arm.	Angle Between Crank and Eccentric.	Position of Eccentric Relative to Crank.
I	Direct	Direct	$90° +$ angle of advance	Ahead of crank
II	Direct	Reversing	$90° -$ angle of advance	Behind crank
III	Indirect	Direct	$90° -$ angle of advance	Behind crank
IV	Indirect	Reversing	$90° +$ angle of advance	Ahead of crank

The above table may be applied equally well whichever direction the engine runs. It is simply necessary to remember that to set the eccentric *ahead* of the crank is to set it so that it reaches a given point in its revolution before the crank reaches the same point in its revolution.

STRENGTH OF RIVETED

Thickness of Material Required.	Greatest Length of Sections Allowable, 5 Feet.				Greatest Length of Sections Allowable, 3 Feet.												
	Least Thickness of Material Allowable.				Least Thickness of Material Allowable.												
	.18	.20	.21	.21	.22	.22	.23	.24	.25	.26	.27	.28	.29	.30	.31	.32	.33
	Diameter of Flues.																
	Over 6 and not over 7 in.	Over 7 and not over 8 in.	Over 8 and not over 9 in.	Over 9 and not over 10 in	Over 10 and not over 11 in.	Over 11 and not over 12 in.	Over 12 and not over 13 in.	Over 13 and not over 14 in.	Over 14 and not over 15 in.	Over 15 and not over 16 in.	Over 16 and not over 17 in.	Over 17 and not over 18 in.	Over 18 and not over 19 in.	Over 19 and not over 20 in	Over 20 and not over 21 in.	Over 21 and not over 22 in.	Over 22 and not over 23 in.
	Pounds Pressure Allowable.																
.18	189	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
.19	194	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
.20	199	184	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
.21	204	189	179	174	...	...	...	...	...	...	...	...	...	...	...	...	...
.22	...	194	184	179	172	158	...	...	...	...	...	...	...	...	...	...	...
.23	...	199	189	184	179	165	152	...	...	...	...	...	...	...	...	...	...
.24	...	204	194	189	187	172	158	147	...	...	...	...	...	...	...	...	...
.25	...	...	199	194	195	179	165	153	143	...	...	...	...	...	...	...	...
.26	...	...	204	199	203	186	172	159	148	139	...	...	...	...	...	...	...
.27	...	...	...	204	211	193	178	165	154	145	136	...	...	...	...	...	...
.28	...	...	...	...	...	200	185	172	160	150	141	134	...	...	...	...	...
.29	...	...	...	...	...	207	191	178	166	155	146	138	131	...	...	...	...
.30	...	...	...	...	...	...	198	184	172	161	152	143	135	129	...	...	...
.31	...	...	...	...	...	...	205	190	178	166	156	148	140	133	126	...	...
.32	...	...	...	...	...	...	...	196	183	171	161	153	145	137	131	125	...
.33	...	...	...	...	...	...	...	202	189	177	167	157	[illegible]	141	135	[illegible]	122
.34	...	...	...	...	...	...	...	...	194	183	172	162	154	[illegible]	139	132	126
.35	...	...	...	...	...	...	...	...	200	187	177	167	159	150	143	137	130
.36	...	...	...	...	...	...	...	...	...	193	182	172	[illegible]	154	147	141	134
.37	...	...	...	...	...	...	...	...	...	...	187	177	168	159	151	145	138
.38	...	...	...	...	...	...	...	...	...	...	...	182	172	163	155	149	142
.39	...	...	...	...	...	...	...	...	...	...	...	...	176	167	159	152	146
.40	...	...	...	...	...	...	...	...	...	...	...	...	...	172	163	156	150
.41	...	...	...	...	...	...	...	...	...	...	...	...	...	...	167	160	153
.42	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	164	157
.43	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	160

OR LAP-WELDED FLUES.

Greatest Length of Sections Allowable, 30 Inches.

Thickness of Material Required.	Least Thickness of Material Allowable.																
	.34	.35	.36	.37	.38	.39	.40	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
	Diameter of Flues.																
	Over 23 and not over 24 in.	Over 24 and not over 25 in.	Over 25 and not over 26 in.	Over 26 and not over 27 in.	Over 27 and not over 28 in.	Over 28 and not over 29 in.	Over 29 and not over 30 in.	Over 30 and not over 31 in.	Over 31 and not over 32 in.	Over 32 and not over 33 in.	Over 33 and not over 34 in.	Over 34 and not over 35 in.	Over 35 and not over 36 in.	Over 36 and not over 37 in.	Over 37 and not over 38 in.	Over 38 and not over 39 in.	Over 39 and not over 40 in.
	Pounds Pressure Allowable.																
.34	121	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
.35	125	120	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
.36	129	123	119	...	...	...	...	...	...	...	...	...	...	...	...	...	...
.37	132	127	122	117	...	...	...	...	...	...	...	...	...	...	...	...	...
.38	136	130	125	121	116	...	...	...	...	...	...	...	...	...	...	...	...
.39	139	133	129	124	119	115	...	...	...	...	...	...	...	...	...	...	...
.40	143	137	132	127	123	118	115	...	...	...	...	...	...	...	...	...	...
.41	147	141	135	130	126	121	118	114	...	...	...	...	...	...	...	...	...
.42	151	144	139	133	129	124	120	117	112	...	...	...	...	...	...	...	...
.43	154	147	142	137	132	127	123	119	115	112	...	...	...	...	...	...	...
.44	157	151	145	140	135	130	126	122	118	115	110	...	...	...	...	...	...
.45	...	155	148	143	138	133	129	124	121	117	113	110	...	...	...	...	...
.46	...	...	151	146	141	136	132	127	124	120	116	112	109	...	...	...	...
.47	...	...	...	[illegible]	144	139	135	130	126	122	118	115	112	109	...	...	...
.48	...	...	...	...	147	142	138	133	129	125	120	117	114	111	108	...	...
.49	...	...	...	...	...	145	140	135	131	127	123	120	116	113	111	108	...
.50	...	...	...	...	...	...	143	138	[illegible]	130	126	122	119	116	113	110	107
.51	...	...	...	...	...	...	...	141	137	133	129	125	121	118	115	112	109
.52	...	...	...	...	...	...	...	...	140	135	131	127	123	120	117	114	111
.53	...	...	...	...	...	...	...	...	...	138	134	129	126	123	119	116	113
.54	...	...	...	...	...	...	...	...	...	...	137	132	128	125	122	118	116
.55	...	...	...	...	...	...	...	...	...	...	...	135	131	127	124	121	118
.56	...	...	...	...	...	...	...	...	...	...	...	...	133	130	126	123	120
.57	...	...	...	...	...	...	...	...	...	...	...	...	...	132	128	125	122
.58	...	...	...	...	...	...	...	...	...	...	...	...	...	...	131	127	124
.59	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	130	126
.60		...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	[illegible]

SPACING OF LINE-SHAFT BEARINGS.

Diameter of Shaft in Inches.	Distance Between Bearings in Feet.	
	Wrought-Iron Shaft.	Steel Shaft.
2	11	11.5
3	13	13.75
[illegible]	15	15.75
5	17	18.25
6	19	20.
7	21	22.25
[illegible]	23	24.
9	25	26.

CONSTANTS FOR APPARENT CUT-OFFS USED IN DETERMINING M. E. P.

Cut-off.	Constant.	Cut-off.	Constant.	Cut-off.	Constant.
$\frac{1}{6}$	.566	$\frac{3}{8}$	.771	[illegible]	.917
$\frac{1}{5}$	.603	.4	.789	.7	.926
$\frac{1}{4}$	.659	$\frac{1}{2}$	.847	$\frac{3}{4}$	.937
.3	.708	.6	.895	.8	.944
$\frac{1}{3}$	.743	$\frac{5}{8}$	.904	$\frac{7}{8}$	.951

EVAPORATIVE POWER OF COAL.

The evaporation per pound of coal for Scotch and fire-box boilers may be taken as follows:

Coal per hour per sq. ft. of grate area. From	6–10	10–14	14–18	18–20
Pounds of water evaporated per pound of coal	10	9	8.25	7.5

For water-tube boilers add $\frac{1}{2}$ pound to the evaporation.

THE THICKNESS OF BOILER TUBES.

The thickness of boiler tubes, as required by the Board of Supervising Inspectors, is given in the table below:

Outside Diameter.	Thickness.	Outside Diameter.	Thickness.
1	.072	$4\frac{1}{2}$	.134
$1\frac{1}{4}$	.072	5	.148
$1\frac{1}{2}$	.083	6	.165
$1\frac{3}{4}$	.095	7	.165
2	.095	8	.165
$2\frac{1}{4}$	.095	9	.180
$2\frac{1}{2}$	.109	10	.203
$2\frac{3}{4}$	.109	11	.220
3	.109	12	.229
$3\frac{1}{4}$	.120	13	.238
$3\frac{1}{2}$	.120	14	.248
$3\frac{3}{4}$	.120	15	.259
4	.134	16	.270

SATURATION AND BOILING POINTS.

Saturation.	Boiling Point.	Saturation.	Boiling Point.
$\frac{0}{32}$	212.0° F.	$\frac{7}{32}$	220.3° F.
$\frac{1}{32}$	213.2° F.	$\frac{8}{32}$	221.5° F.
$\frac{2}{32}$	214.4° F.	$\frac{9}{32}$	222.7° F.
$\frac{3}{32}$	215.5° F.	$\frac{10}{32}$	223.8° F.
$\frac{4}{32}$	216.7° F.	$\frac{11}{32}$	225.0° F.
$\frac{5}{32}$	217.9° F.	$\frac{12}{32}$	226.1° F.
$\frac{6}{32}$	219.1° F.		

At $\frac{12}{32}$ saturation the water becomes saturated; that is, it will not dissolve any more matter.

In the above table, the exact boiling points of sea-water at different degrees of saturation are given As water will

boil at a temperature varying with the pressure of the atmosphere, the boiling points given in the table are correct for but one pressure, namely, 30 in. of mercury.

CONSTANTS USED IN CALCULATING SPEED OF VESSELS.

Description of Vessel.	Speed, Knots.	k.
Under 200 feet, fair........	9–10	200
Under 200 feet, fine........	9–10	230
Under 200 feet, fine.......	10–11	210
Under 200 feet, fine........	11–12	200
From 200–250 feet, fair.....	9–11	220
From 200–250 feet, fine	9–11	240
From 200–250 feet, fine	11–12	220
From 250–300 feet, fair.....	9–11	250
From 250–300 feet, fair.....	11–13	220
From 250–300 feet, fine	9–11	260
From 250–300 feet, fine	11–13	240
From 250–300 feet, fine	13–15	200
From 300–400 feet, fair.....	9–11	260
From 300–400 feet, fair.....	11–13	240
From 300–400 feet, fine	11–13	260
From 300–400 feet, fine	13–15	240
From 300–400 feet, fine	15–17	190
Above 400 feet, fine........	15–17	240

To determine whether a vessel is fair or fine, it is usual to compare its displacement in cubic feet with the volume of a rectangular box having a length equal to the length of the vessel on the water-line, a width equal to the beam, and a depth equal to the draught of the vessel diminished by the depth of the keel. If the displacement is .55 of the volume of the box, or less, the vessel is fine; if above .55 and less than .70, fair. The quotient obtained by dividing the displacement by the contents of the imaginary box is called the **coefficient of fineness.**

SATURATION AND MANAGEMENT OF FEED.

The following scheme shows the method of regulating the saturation in a boiler containing 100,000 pounds of sea-water. The saturation is not to exceed $\frac{3}{32}$:

	Number of Pounds of Water in Boiler.	Pounds of Solid Matter.
Evaporated into steam.......	100,000	3,125
Fed in to make up at $\frac{1}{32}$.....	100,000	3,125
Water in boiler..............	100,000	6,250
Blown off....................	20,000	1,250
Water in boiler..............	80,000	5,000
Fed in at $\frac{1}{32}$................	20,000	625
Water in boiler..............	100,000	5,625
Evaporated..................	20,000	Steam
Water in boiler..............	80,000	5,625
Fed in at $\frac{1}{32}$................	20,000	625
Water in boiler..............	100,000	6,250

FORMULAS.

FORMULAS USED IN MENSURATION.

THE PARALLELOGRAM.

h = altitude of parallelogram, expressed in any unit;
b = base of parallelogram, expressed in same unit;
A = area of parallelogram.

$$A = hb. \qquad \textbf{(44.)} \quad \text{Art. } \textbf{375.}$$

$$\left.\begin{aligned} h &= \frac{A}{b}. \\ b &= \frac{A}{h}. \end{aligned}\right\} \qquad \textbf{(46.)} \quad \text{Art. } \textbf{377.}$$

THE TRAPEZOID.

h = altitude of a trapezoid;
l = length of one of its parallel sides;
l_1 = length of the other parallel side;
A = area of trapezoid.

$$A = h\left(\frac{l+l_1}{2}\right). \qquad \textbf{(45.)} \quad \text{Art. } \textbf{376.}$$

THE TRIANGLE.

A, B, C = the number of degrees in the three angles, respectively.

$$\left.\begin{aligned} A &= 180^\circ - B - C. \\ B &= 180^\circ - A - C. \\ C &= 180^\circ - B - A. \end{aligned}\right\} \qquad \textbf{(47.)} \quad \text{Art. } \textbf{382.}$$

A_1 and B_1 = the number of degrees in the two acute angles, respectively, of any right-angled triangle.

$$\left.\begin{aligned} A_1 &= 90^\circ - B_1. \\ B_1 &= 90^\circ - A_1. \end{aligned}\right\} \qquad \textbf{(48.)} \quad \text{Art. } \textbf{383.}$$

a and b = the lengths, respectively, of the two short sides of a right-angled triangle;
c = length of third side, or hypotenuse.

$$c = \sqrt{a^2 + b^2}. \qquad \textbf{(49.)} \quad \text{Art. } \mathbf{385.}$$

$$\left.\begin{aligned} a &= \sqrt{c^2 - b^2}. \\ b &= \sqrt{c^2 - a^2}. \end{aligned}\right\} \qquad \textbf{(50.)} \quad \text{Art. } \mathbf{385.}$$

h = altitude of given triangle;
b = base of triangle;
A = area of triangle.

$$A = \frac{bh}{2}. \qquad \textbf{(51.)} \quad \text{Art. } \mathbf{386.}$$

$$\left.\begin{aligned} h &= \frac{2A}{b}. \\ b &= \frac{2A}{h}. \end{aligned}\right\} \qquad \textbf{(52.)} \quad \text{Art. } \mathbf{386.}$$

THE POLYGON.

N = number of sides in any regular polygon.
D = number of degrees in each interior angle.

$$D = \frac{180\,(N-2)}{N}. \qquad \textbf{(53.)} \quad \text{Art. } \mathbf{389.}$$

l = length of side of any regular polygon;
d = perpendicular distance from center of polygon (i.e. center of circumscribing circle) to any side;
N = number of sides;
A = area of polygon.

$$A = \frac{dlN}{2}. \qquad \textbf{(54.)} \quad \text{Art. } \mathbf{390.}$$

THE CIRCLE.

d = diameter of circle;
c = circumference;
A = area;
l = length of arc;
n = number of degrees in arc;
a = area of sector;

a_1 = area of segment;
n_1 = number of degrees in sector;
π = 3.1416.

$c = \pi d.$ (55.) Art. **402.**

$d = \frac{c}{\pi}.$ (56.) Art. **403.**

$l = \frac{cn}{360}.$ (57.) Art. **404.**

$A = \frac{\pi d^2}{4} = .7854 d^2.$ (58.) Art. **405.**

$d = \sqrt{\frac{A}{.7854}}.$ (59.) Art. **406.**

$a = \frac{n_1 A}{360}.$ (60.) Art. **407.**

$a_1 = \frac{4h^2}{3}\sqrt{\frac{d}{h} - .608.}$ (61.) Art. **408.**

THE PRISM, CYLINDER, CONE, AND PYRAMID.

p = perimeter of the base of the prism, cylinder, cone, or pyramid;
h = altitude;
h_1 = slant height of cone;
A = area of convex surface;
a = area of base;
A_1 = total area of outside surface;
V = volume.

Prism and cylinder $\begin{cases} A = ph. \\ A_1 = A + 2a. \\ V = ah. \end{cases}$ (62.) Art. **416.** (63.) Art. **417.**

Pyramid and cone $\begin{cases} A = \frac{ph_1}{2}. \\ A_1 = A + a. \\ V = \frac{ah}{3}. \end{cases}$ (64.) Art. **422.** (65.) Art. **423.**

FRUSTUM OF CONE OR PYRAMID.

p = perimeter of upper base of frustum;
p_1 = perimeter of lower base of frustum;
h = altitude of frustum;
h_1 = slant height;
a = area of upper base;
a_1 = area of lower base;
A = convex surface;
A_1 = total surface;
V = volume.

$$\left.\begin{aligned} A &= \left(\frac{p+p_1}{2}\right)h_1. \\ A_1 &= A + a + a_1. \end{aligned}\right\} \qquad (66.) \quad \text{Art. } \mathbf{426.}$$

$$V = (a + a_1 + \sqrt{a a_1})\frac{h}{3} \qquad (67.) \quad \text{Art. } \mathbf{427.}$$

THE SPHERE.

d = diameter of sphere;
A = area of surface;
V = volume;
π = 3.1416.

$$A = \pi d^2. \qquad (68.) \quad \text{Art. } \mathbf{429.}$$

$$V = \frac{1}{6}\pi d^3 = .5236\, d^3. \qquad (69.) \quad \text{Art. } \mathbf{430.}$$

THE CYLINDRICAL RING.

a = area of cross-section of ring;
c = circumference of cross-section;
D = mean circumference of ring;
A = convex area of ring;
V = volume of ring.

$$A = Dc. \qquad (70.) \quad \text{Art. } \mathbf{431.}$$

$$V = Da. \qquad (71.) \quad \text{Art. } \mathbf{432.}$$

FORMULAS USED IN MECHANICS.

MOTION AND VELOCITY.

s = distance traveled by moving body;
v = uniform velocity of body;
t = the time.

$$v = \frac{s}{t}. \qquad (72.) \quad \text{Art. } \mathbf{465.}$$

$$s = v\,t. \qquad (73.) \quad \text{Art. } \mathbf{467.}$$

$$t = \frac{s}{v}. \qquad (74.) \quad \text{Art. } \mathbf{468.}$$

CENTER OF GRAVITY.

w = weight of smaller body;
W = weight of larger body;
l = distance between centers of gravity of the two bodies;
l_1 = distance from the center of gravity of the two to center of gravity of larger body.

$$l_1 = \frac{w\,l}{W+w}. \qquad (75.) \quad \text{Art. } \mathbf{478.}$$

THE LEVER.

P = the power;
W = the weight;
a = perpendicular distance of power from fulcrum = power arm;
b = perpendicular distance of weight from fulcrum = weight arm;
$a_1, a_2, a_3, \ldots$ = power arms of compound lever;
$b_1, b_2, b_3, \ldots$ = weight arms of compound lever.

$$P a = W b. \qquad (79.) \quad \text{Art. } \mathbf{486.}$$

$$P \times a_1 \times a_2 \times a_3 \times \ldots\ldots = W \times b_1 \times b_2 \times b_3 \times \ldots\ldots \qquad (80.)$$

Art. **490.**

SPEED AND POWER OF PULLEYS.

D = diameter of driving pulley;
d = diameter of driven pulley;
N = revolutions per minute of driving pulley;
n = revolutions per minute of driven pulley.

$$D = \frac{dn}{N}. \qquad (81.) \quad \text{Art. } \mathbf{496.}$$

$$d = \frac{DN}{n}. \qquad (82.) \quad \text{Art. } \mathbf{497.}$$

$$n = \frac{DN}{d}. \qquad (83.) \quad \text{Art. } \mathbf{498.}$$

$$N = \frac{dn}{D}. \qquad (84.) \quad \text{Art. } \mathbf{499.}$$

$D_1, D_2, D_3, \ldots$ = diameters of driving pulleys;
$d_1, d_2, d_3, \ldots$ = diameters of driven pulleys.
P = power exerted;
W = weight lifted

$$\left.\begin{aligned} P &= \frac{W \times d_1 \times d_2 \times d_3 \times \ldots}{D_1 \times D_2 \times D_3 \times \ldots}. \\ W &= \frac{P \times D_1 \times D_2 \times D_3 \times \ldots}{d_1 \times d_2 \times d_3 \times \ldots}. \end{aligned}\right\} \; (85.) \quad \text{Art. } \mathbf{501}$$

GEAR-WHEELS.

D = pitch diameter of wheel;
P = pitch;
T = number of teeth.

$$D = \frac{PT}{3.1416}. \qquad (86.) \quad \text{Art. } \mathbf{508.}$$

$$T = \frac{3.1416\,D}{P}. \qquad (87.) \quad \text{Art. } \mathbf{508.}$$

$$P = \frac{3.1416\,D}{T}. \qquad (88.) \quad \text{Art. } \mathbf{508.}$$

N = revolutions per minute of driving wheel;
n = revolutions per minute of driven wheel;
T = number of teeth in the driving wheel;
t = number of teeth in the driven wheel.

$$T = \frac{t\,n}{N}. \qquad (89.) \quad \text{Art. } \mathbf{511.}$$

$$t = \frac{T\,N}{n}. \qquad (90.) \quad \text{Art. } \mathbf{511.}$$

$$n = \frac{T\,N}{t}. \qquad (91.) \quad \text{Art. } \mathbf{511.}$$

$$N = \frac{t\,n}{T}. \qquad (92.) \quad \text{Art. } \mathbf{511.}$$

PULLEYS.

n = number of parts of rope supporting load; one continuous rope being used, and free end not counted;

P = power;

W = weight lifted.

$$\left.\begin{aligned} W &= P\,n. \\ P &= \frac{W}{n}. \end{aligned}\right\} \qquad (93.) \quad \text{Art. } \mathbf{516.}$$

THE INCLINED PLANE.

l = length of inclined plane;

b = base of plane;

h = height of plane;

P = power;

W = weight.

When the power acts parallel to the plane:

$$P = \frac{W\,h}{l}; \quad W = \frac{P\,l}{h}. \qquad (94.) \quad \text{Art. } \mathbf{518.}$$

When the power acts parallel to the base:

$$P = \frac{W\,h}{b}; \quad W = \frac{P\,b}{h}. \qquad (95.) \quad \text{Art. } \mathbf{518.}$$

THE SCREW.

p = pitch of screw;

r = perpendicular distance from center of screw to line of direction of power;

P = power;

W = weight.

$$\left.\begin{aligned} P &= \frac{Wp}{6.2832\,r} = \frac{Wp}{2\pi r}. \\ W &= \frac{6.2832\,Pr}{p} = \frac{2\pi r P}{p}. \end{aligned}\right\} \qquad (96.) \quad \text{Art. } \mathbf{521.}$$

FRICTION ON GUIDES.

P = total pressure on piston;
l = length of main rod;
r = length of crank;
c = coefficient of friction between cross-head and guides;
F = effort required to overcome friction.

$$F = \frac{Prc}{l}. \qquad (97.) \quad \text{Art. } \mathbf{532.}$$

CENTRIFUGAL FORCE.

F = centrifugal force in pounds;
W = weight of revolving body in pounds;
R = radius of circle described by body in feet;
N = revolutions per minute.

$$F = .00034\,WRN^2. \qquad (98.) \quad \text{Art. } \mathbf{535.}$$

WORK AND ENERGY.

F = force required to overcome the resistance;
S = space through which the resistance acts;
U = work done in foot-pounds;
T = time occupied in minutes;
H = horsepower;
W = weight of a body;
h = vertical distance the body is raised.

$$U = FS = Wh. \qquad (99.) \quad \text{Art. } \mathbf{541.}$$

$$H = \frac{FS}{33{,}000\,T} = \frac{Wh}{33{,}000\,T}. \quad \text{Art. } \mathbf{543.}$$

W = weight of moving body in pounds;
v = velocity of body in feet per second;
E_k = kinetic energy of body in foot-pounds.

$$E_k = \frac{Wv^2}{64.32}. \qquad (100.) \quad \text{Art. } \mathbf{545.}$$

BELTS.

D = diameter of one pulley;
D_1 = diameter of other pulley;
L = distance between shafts;
B = length of *open* belt.

$$B = 3\tfrac{1}{4}\left(\frac{D+D_1}{2}\right) + 2L. \qquad \textbf{(101.)} \quad \text{Art. } \textbf{551.}$$

W = width of single belt in inches;
W_1 = width of double belt in inches;
H = horsepower transmitted by belt;
S = speed of belt in feet per minute.

$$W = \frac{800\,H}{S}. \qquad \textbf{(102.)} \quad \text{Art. } \textbf{552.}$$

$$H = \frac{W\,S}{800}. \qquad \textbf{(103.)} \quad \text{Art. } \textbf{553.}$$

$$W_1 = \frac{2}{3}\,W. \qquad \textbf{(104.)} \quad \text{Art. } \textbf{555.}$$

HORSEPOWER OF GEARS.

p = pitch of teeth of gear (breadth of face is $2\frac{1}{2}$ to $3p$):
s = speed of point on pitch circle in feet per minute;
H = horsepower transmitted by gear.

$$H = .01\,s\,p^2. \qquad \textbf{(105.)} \quad \text{Art. } \textbf{557.}$$

LIQUID PRESSURE.

a = area of a submerged surface in square inches;
d = distance in inches of center of gravity of surface from surface of liquid;
w = weight of a cubic inch of the fluid in pounds;
p = pressure on surface of liquid, pounds per sq. in.;
P = total pressure on submerged surface in pounds.

$$P = a\,(d\,w + p). \quad \textbf{(106.) (107.) (108.)} \quad \text{Arts. } \textbf{565-572.}$$

VOLUME AND PRESSURE OF GASES.

p = original pressure;
v = original volume;
p_1 = final pressure;
v_1 = final volume.

$$p_1 = \frac{p\,v}{v_1}. \qquad \textbf{(109.)} \quad \text{Art. } \textbf{594.}$$

$$v_1 = \frac{p\,v}{p_1}. \qquad \textbf{(110.)} \quad \text{Art. } \textbf{595.}$$

STRENGTH OF BARS IN TENSION.

W = safe load in pounds;
A = area of minimum cross-section in square inches;
S = working stress in pounds per square inch (see table of Tensile Strengths of Materials).

$$W = A\,S. \qquad \textbf{(111.)} \quad \text{Art. } \textbf{626.}$$

$$A = \frac{W}{S}. \qquad \textbf{(112.)} \quad \text{Art. } \textbf{626.}$$

$$S = \frac{W}{A}. \qquad \textbf{(113.)} \quad \text{Art. } \textbf{626.}$$

STRENGTH OF CHAINS.

W = safe load in pounds for stud-link chains;
W_1 = safe load in pounds for close-link chains;
D = diameter of iron from which link is made, inches.

$$W = 18{,}000\,D^2. \qquad \textbf{(114.)} \quad \text{Art. } \textbf{628.}$$

$$W_1 = 12{,}000\,D^2. \qquad \textbf{(115.)} \quad \text{Art. } \textbf{628.}$$

STRENGTH OF ROPES.

C = circumference of **hemp** rope in inches;
W = maximum working load in pounds.

$$W = 100\,C^2. \qquad \textbf{(116.)} \quad \text{Art. } \textbf{630.}$$

$$C = .1\sqrt{W}. \qquad \textbf{(117.)} \quad \text{Art. } \textbf{630.}$$

C = circumference of **wire** rope in inches;
W = maximum working load in pounds;
k = constant: 600 for iron; 1,000 for steel;
c = constant: .0408 for iron; .0316 for steel.

$$W = k C^2. \qquad (118.) \quad \text{Art. } \mathbf{633.}$$

$$C = c \sqrt{W}. \qquad (119.) \quad \text{Art. } \mathbf{633.}$$

STRENGTH OF PILLARS.

C = crushing strength in tons per sq. in. (see table of Crushing Strengths of Materials);
S = sectional area in inches;
L = length in inches;
d = least thickness of rectangular pillar, or diameter of round pillar, in inches;
W = breaking load in tons;
a = constant (see table of Constants for Pillars).

$$W = \frac{C S}{1 + \frac{L^2}{a d^2}}. \qquad (121.) \quad \text{Art. } \mathbf{638.}$$

STRENGTH OF BEAMS.

d = depth of beam in inches;
w = width of beam in inches;
d_1 = diameter of cylindrical beam in inches;
L = length between supports in feet
= distance between load and fixed end, in the case of cantilevers.
S = safe transverse strength (see table of Constants for Transverse Strength of Beams).
W = safe load in pounds.

Cantilevers. (Load at End.)

$$W = \frac{d^2 w S}{L}. \qquad (122.) \quad \text{Art. } \mathbf{642.}$$

$$W = \frac{.6 d_1^{\,3} S}{L}. \qquad (123.) \quad \text{Art. } \mathbf{643.}$$

If the load is distributed uniformly, multiply the results obtained from formulas **122** and **123** by 2.

Beams Supported at the Ends.

$$W = \frac{4\,d^2\,w\,S}{L}. \qquad \textbf{(124.)} \quad \text{Art. } \textbf{644.}$$

$$W = \frac{4\,d_1^2 \times .6\,S}{L}. \qquad \textbf{(125.)} \quad \text{Art. } \textbf{645.}$$

If the load is uniformly distributed, multiply the results obtained by 2.

SHEARING STRENGTH OF MATERIALS.

a = area of cross-section in square inches;
S = safe shearing stress (see table of Shearing Strengths of Materials);
W = safe load in pounds.

$$W = a\,S. \qquad \textbf{(126.)} \quad \text{Art. } \textbf{649.}$$

LINE SHAFTING.

D = diameter of shaft;
R = revolutions per minute;
H = horsepower transmitted;
C = constant (see table of Constants for Line Shafting)

$$H = \frac{D^3\,R}{C}. \qquad \textbf{(127.)} \quad \text{Art. } \textbf{653.}$$

$$R = \frac{C\,H}{D^3}. \qquad \textbf{(128.)} \quad \text{Art. } \textbf{654.}$$

$$D = \sqrt[3]{\frac{C\,H}{R}}. \qquad \textbf{(129.)} \quad \text{Art. } \textbf{655.}$$

FORMULAS USED IN STEAM AND STEAM BOILERS.

SPECIFIC HEAT.

W = weight of body in pounds;
t = temperature before heat is applied;
t_1 = temperature after heat is applied;
c = specific heat of body;
U = number of B. T. U. required to raise temperature of body from t to t_1.

$$U = c\,W(t_1 - t). \qquad \textbf{(130.)} \quad \text{Art. } \textbf{667.}$$

TEMPERATURE OF MIXTURES.

$w, w_1, w_2, \ldots$ = weights of the several substances, respectively;
$c, c_1, c_2, \ldots$ = specific heats of the substances, respectively;
$t, t_1, t_2, \ldots$ = temperatures of the substances, respectively;
T = final temperature of mixture.

$$T = \frac{w\,c\,t + w_1 c_1 t_1 + w_2 c_2 t_2 + \ldots}{w\,c + w_1 c_1 + w_2 c_2 + \ldots}. \qquad \textbf{(131.)} \quad \text{Art. } \textbf{670.}$$

Mixture of Steam and Water.

W = weight of steam in pounds;
w = weight of water in pounds;
t_1 = temperature of steam;
t = temperature of water;
T = final temperature of mixture;
L = latent heat of steam at the given temperature.

$$T = \frac{W(L + t_1) + w\,t}{W + w}. \qquad \textbf{(132.)} \quad \text{Art. } \textbf{671.}$$

PROPORTIONS OF RIVETED JOINTS.

p = pitch of rivets in inches;
p_m = maximum pitch allowable;
n = number of rows of rivets;
p_d = diagonal pitch in inches;
d = diameter of rivets in inches;
T = thickness of plate in inches;
V = distance between rows of rivets in inches;
E = distance from center of rivet to edge of plate in inches.

When plate and rivets are iron, for single-riveted lap joints:

$$d = T + \tfrac{3}{8}''. \qquad (132a.) \quad \text{Art. } \mathbf{716}b.$$

For double-riveted lap joints:

$$d = T + \tfrac{5}{16}''. \qquad (132b.) \quad \text{Art. } \mathbf{716}b.$$

When plate and rivets are steel, for single-riveted lap joints:

$$d = T + \tfrac{7}{16}''. \qquad (132c.) \quad \text{Art. } \mathbf{716}b.$$

For double-riveted lap joints:

$$d = T + \tfrac{3}{8}''. \qquad (132d.) \quad \text{Art. } \mathbf{716}b.$$

For iron plates and iron rivets:

$$p = \frac{.7854\, d^2 n}{T} + d. \qquad (132e.) \quad \text{Art. } \mathbf{716}b.$$

For steel plates and steel rivets:

$$p = \frac{23 \times .7854\, d^2 n}{28\, T} + d. \qquad (132f.) \quad \text{Art. } \mathbf{716}b.$$

For all kinds of joints and material:

$$E = 1.5\, d. \qquad (132g.) \quad \text{Art. } \mathbf{716}b.$$

For double chain-riveted lap joints:

$$V = \frac{4\,d + 1}{2}. \qquad (132h.) \quad \text{Art. } \mathbf{716}b.$$

For double zigzag-riveted lap joints:

$$V = \frac{\sqrt{(11\,p + 4\,d)\,(p + 4\,d)}}{10}. \qquad (132i.) \quad \text{Art. } \mathbf{716}b.$$

$$p_a = \frac{6p + 4d}{10}. \qquad \textbf{(132}j.\textbf{)} \quad \text{Art. } \textbf{716}c.$$

For a single-riveted lap joint:

$$p_m = 1.31\,T + 1\tfrac{5}{8}''. \qquad \textbf{(132}k.\textbf{)} \quad \text{Art. } \textbf{716}c.$$

For a double-riveted lap joint:

$$p_m = 2.62\,T + 1\tfrac{5}{8}''. \qquad \textbf{(132}l.\textbf{)} \quad \text{Art. } \textbf{716}c.$$

STRESS IN STAYS.

A = area in square inches supported by a stay (whether screw stay, staybolt, or stayrod);

B = the cross-sectional area of the stay in square inches (corresponding to diameter at bottom of the thread, if the stay is screwed);

P = steam pressure in pounds per square inch;

L = load on the stay in pounds;

S = stress per square inch of cross-section of the stay;

S_t = total allowable stress on a stay;

M = legal stress per square inch of cross-section of the stay;

A_t = total area to be supported in square inches;

N = number of stays.

$$L = A\,P. \qquad \textbf{(133.)} \quad \text{Art. } \textbf{728.}$$

$$S = \frac{A\,P}{B}. \qquad \textbf{(134.)} \quad \text{Art. } \textbf{728.}$$

$$S_t = B\,M. \qquad \textbf{(134}a.\textbf{)} \quad \text{Art. } \textbf{728.}$$

$$P = \frac{B\,M}{A}. \qquad \textbf{(135.)} \quad \text{Art. } \textbf{728.}$$

$$N = \frac{A_t\,P}{S_t}. \qquad \textbf{(135}a.\textbf{)} \quad \text{Art. } \textbf{728.}$$

PALM STAYS.

b = distance in inches from boiler head to intersection of the center line of the palm stay with the shell;

l = length of the palm stay in inches, measured between head and shell along the center line of the stay;

A = area supported by the stay in square inches;
P = steam pressure in pounds;
M = legal stress per square inch of cross-section of the stay.

$$L = \frac{l}{b} A P. \qquad \textbf{(135}b\textbf{.)} \quad \text{Art. } \textbf{729}a.$$

$$P = \frac{b}{l} \times \frac{B M}{A}. \qquad \textbf{(135}c\textbf{.)} \quad \text{Art. } \textbf{729}c.$$

STRENGTH OF STAYED SURFACES.

P = working pressure in pounds per square inch;
t = thickness of plate in sixteenths of an inch;*
a = pitch of stays (arranged as in Fig. 186, Art. **727**).

When fitted with either screw stays, staybolts and nuts, or socket staybolts, and the surfaces are those of fire-boxes, furnaces, or back connections, we have:

When the plates do not exceed $\frac{7}{16}''$ in thickness:

$$P = \frac{112\, t^2}{a^2}. \qquad \textbf{(136.)} \quad \text{Art. } \textbf{730.}$$

$$a = \sqrt{\frac{112\, t^2}{P}}. \qquad \textbf{(138.)} \quad \text{Art. } \textbf{731.}$$

And when the plates exceed $\frac{7}{16}''$ in thickness:

$$P = \frac{120\, t^2}{a^2}. \qquad \textbf{(137.)} \quad \text{Art. } \textbf{730.}$$

$$a = \sqrt{\frac{120\, t^2}{P}}. \qquad \textbf{(138.)} \quad \text{Art. } \textbf{731.}$$

When the surfaces are other than those mentioned above, and stayrods with screwed ends are used, having nuts inside and out, as in Fig. 180, Art. **722**, we have

$$P = \frac{140\, t^2}{a^2}. \qquad \textbf{(139.)} \quad \text{Art. } \textbf{732.}$$

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*NOTE.—In the following formulas—**136** to **140**—the symbol t^2 means "the square of the number of sixteenths contained in the thickness of the plate." Thus, if the plate is $\frac{3}{8}''$ thick, there are 6 sixteenths in its thickness, and, therefore, $t^2 = 6^2 = 36$.

When the conditions are the same as for formula **139**, except that the stayrods are provided with a square washer riveted to the head and having a thickness not less than half that of the plate, and the length of the side not less than seven-eighths of the pitch of the stayrod, formula **140** may be used, taking t at 80% of the combined thickness of plate and washer. Also, when a stiffening plate equal in thickness to at least half that of the head is used and securely riveted to the head, and when double angle irons riveted to the plate with stayrods between them and legs having a thickness not less than two-thirds that of the plate, and a depth of not less than one-quarter of the pitch of the stayrods, are used, 80% of the combined thickness is to be taken as the value of t; the working pressure allowed is then given by

$$P = \frac{200\,t^2}{a^2}. \qquad \textbf{(140.)} \quad \text{Art. } \mathbf{733}.$$

BOILER FLUES.

Smoke Flues.

P = working pressure in pounds per square inch, given in Table 27, corresponding to diameter and thickness of flue;

L = corresponding length of sections of flue in above table;

P_1 = the required working pressure in pounds per square inch;

L_1 = actual length of sections of flue.

NOTE.—L and L_1 are to be measured between centers of circular seams.

Then, $$P_1 = \frac{PL}{L_1}. \qquad \textbf{(141.)} \quad \text{Art. } \mathbf{736}.$$

Furnace Flues.

D = diameter of flue in inches;

T = thickness of flue in decimals of an inch;

L = length of flue in feet (not to exceed 8 feet);

P = working pressure in pounds per square inch.

$$P = \frac{89{,}600\,T^2}{LD}. \qquad \textbf{(142.)} \quad \text{Art. } \mathbf{743}.$$

In furnaces made up in sections with flanged ends [see Fig. 190 (A), Art. **740**], L is to be taken as the length of each section, if not more than 8 feet; where each section is strengthened at equal intervals by angle-iron rings [Fig. 190 (B)], L is to be taken as the length of one of the portions into which the section is thus divided. Thus, if each section were $6\frac{1}{2}$ feet long, and a strengthening ring were fixed midway between the ends, the value of L would be taken as $3\frac{1}{4}$ feet. If the section were 10 feet long and two rings were used (equally spaced), L would be $\frac{10}{3} = 3\frac{1}{3}$ feet.

When the flues are strengthened with half-round rings [Fig. 190 (C)], we have

$$P = \frac{67{,}200\,T^2}{L\,D}. \qquad \textbf{(143.)} \quad \text{Art. } \textbf{743.}$$

L is to be taken in the same way as in previous rule.

Corrugated and Ribbed Furnace Flues.

T = thickness of flue in decimals of an inch;
D = mean diameter of flue in inches;
P = working pressure in pounds per square inch.

$$P = \frac{14{,}000\,T}{D}. \qquad \textbf{(144.)} \quad \text{Art. } \textbf{746.}$$

NOTE.—For corrugated flues D is to be taken as the outside diameter measured over bottom of corrugations, and in the case of ribbed flues, D is to be measured (outside) over the flat part of flue.

MORISON SUSPENSION FLUE.

The symbols having the same meaning as in the previous rule, and taking for D the outside diameter over the bottom of the corrugations, we have

$$P = \frac{15{,}000\,T}{D}. \qquad \textbf{(144}a\textbf{.)} \quad \text{Art. } \textbf{746}a\textbf{.}$$

HEADS OF BOILER OR STEAM DRUMS (UNSTAYED).

Bumped Heads.

P = working pressure in pounds per square inch;
T = thickness of head in inches;
R = radius in inches to which the head is bumped or dished;
A = area of flat head in square inches;
S = tensile strength of material in pounds per square inch.

When the concave or hollow side is subject to the steam pressure (Fig. 193 (*a*), Art. **747**), and the head is single riveted to the shell:

$$P = \frac{TS}{3R}. \qquad \textbf{(145.)} \quad \text{Art. } \textbf{747.}$$

When the head is double riveted to the shell:

$$P = \frac{TS}{2.5R}. \qquad \textbf{(145.)} \quad \text{Art. } \textbf{747.}$$

When the head is reversed, the hollow side being set outwards, away from the steam [Fig. 193 (*b*)], and the head is single riveted to the shell:

$$P = \frac{TS}{5R}. \qquad \textbf{(146.)} \quad \text{Art. } \textbf{748.}$$

When the head is double riveted to the shell:

$$P = \frac{TS}{4\frac{1}{4}R}. \qquad \textbf{(146.)} \quad \text{Art. } \textbf{748.}$$

For flat unstayed heads:

$$P = \frac{TS}{.54A}. \qquad \textbf{(147.)} \quad \text{Art. } \textbf{749.}$$

When these heads are stayed, the working pressure may be found by means of rules **135***c* and **139.**

BOILER SHELLS.

T = thickness of plate of cylinder or boiler in inches;
R = internal radius in inches;
S = ultimate tensile strength of the material;

P_b = bursting pressure in pounds per square inch;
P = safe working pressure in pounds per square inch.

$$P_b = \frac{T\,S}{R}. \qquad \textbf{(148.)} \quad \text{Art. } \mathbf{752.}$$

For boilers with single-riveted longitudinal seams, the value of P is fixed by law, thus:

$$P = \frac{T\,S}{6\,R}. \qquad \textbf{(149.)} \quad \text{Art. } \mathbf{753.}$$

For boilers with double-riveted longitudinal seams, the value of P by law is:

$$P = \frac{T\,S}{5\,R}. \qquad \textbf{(150.)} \quad \text{Art. } \mathbf{754.}$$

The above rules assume that none of the rivet holes are punched, and also that corresponding holes match fairly without any drifting.

SAFETY VALVES.

A = area of the valve in square inches;
D = distance in inches from center line of valve to fulcrum;
L = distance in inches of weight from fulcrum;
P = steam pressure in pounds per square inch;
W = weight on the lever in pounds;
w = weight of valve and stem in pounds added to the downward pressure due to weight of lever.

NOTE.—By "the area of a safety valve" is meant the area of that part which is exposed to the steam pressure when the valve is seated.

$$P = \frac{\frac{W\,L}{D} + w}{A}. \qquad \textbf{(151.)} \quad \text{Art. } \mathbf{767.}$$

$$L = \frac{(A\,P - w)\,D}{W}. \qquad \textbf{(152.)} \quad \text{Art. } \mathbf{767.}$$

$$W = \frac{(A\,P - w)\,D}{L}. \qquad \textbf{(153.)} \quad \text{Art. } \mathbf{768.}$$

SATURATION AND THE SALINOMETER.

Testing the Water in Boiler.

A = boiling point of water tested;
B = product of the number of tenths of an inch variation in the height of barometer and .16; this product to be subtracted when the barometer is above 30 in., and added when below 30 in.;
S = the degree of saturation of the water. (S will be the numerator of the fraction expressing the saturation.)

$$S = \frac{(A \pm B) - 212}{1.2}. \qquad (154.) \quad \text{Art. } \mathbf{817.}$$

Amount of Water To Be Blown Off.

A = weight of water to be blown off in pounds;
B = numerator of fraction expressing the saturation of A;
C = weight of feed-water in pounds;
D = numerator of fraction expressing the saturation of C;
E = weight of water evaporated, corresponding to C.

$$A = \frac{C\,D}{B}. \qquad (155.) \quad \text{Art. } \mathbf{820.}$$

$$E = A\left(\frac{B}{D} - 1\right). \qquad (156.) \quad \text{Art. } \mathbf{820.}$$

Loss of Heat Due to Blowing Off.

A = total heat (reckoned above the temperature of the feed) imparted to the amount of water converted into steam for 1 pound of water blown off;
B = number of B. T. U. lost in blowing off 1 pound of water;
L = percentage of heat lost.

$$L = \frac{100\,B}{A + B}. \qquad (157.) \quad \text{Art. } \mathbf{822.}$$

COMBUSTION.

Particulars of Combustion.—Evaporation.

C = the number of parts of carbon contained in 100 parts of a given fuel;

H = the number of parts of hydrogen contained in 100 parts of the same fuel;

A = the number of cubic feet of air required for the combustion of a pound of the above fuel;

B = heat of combustion of a pound of the fuel in B. T. U.;

W = number of pounds of water at 212° evaporated by a pound of the above fuel.

$$A = 1.52\,(C + 3\,H). \qquad \textbf{(158.)} \quad \text{Art. } \textbf{838.}$$

$$B = 145\,C + 620\,H. \qquad \textbf{(159.)} \quad \text{Art. } \textbf{839.}$$

$$W = \frac{B}{966}. \qquad \textbf{(160.)} \quad \text{Art. } \textbf{839.}$$

Rate of Combustion Under Natural Draft.

W = maximum weight (in pounds) of coal burned per square foot of grate area per hour;

H = height of smokestack in feet.

NOTE.—The height of smokestack is the perpendicular distance between the top of the stack and the top of the grate.

Anthracite coal burning under the most favorable conditions:

$$W = 2\sqrt{H} - 1. \qquad \textbf{(161.)} \quad \text{Art. } \textbf{853.}$$

Anthracite coal burning under ordinary conditions:

$$W = 1.5\sqrt{H} - 1. \qquad \textbf{(162.)} \quad \text{Art. } \textbf{853.}$$

The best semi-anthracite and bituminous coals:

$$W = 2.25\sqrt{H}. \qquad \textbf{(163.)} \quad \text{Art. } \textbf{853.}$$

Ordinary soft coals:

$$W = 3\sqrt{H}. \qquad \textbf{(164.)} \quad \text{Art. } \textbf{853.}$$

GRATE AREA AND HEATING SURFACE.

G = area of grate in square feet;
H = area of heating surface in square feet;
F = rate of combustion in pounds per square foot of grate area per hour;
W = weight of steam generated by boiler per hour;
E = pounds of water evaporated per pound of coal per hour;
R = ratio of heating surface to grate area; it is from 30 to 35 for Scotch or fire-box boilers, and from 35 to 40 for water-tube boilers.

$$G = \frac{W}{FE}. \qquad \textbf{(165.)} \quad \text{Art. } \mathbf{900.}$$

$$H = RG. \qquad \textbf{(166.)} \quad \text{Art. } \mathbf{901.}$$

EVAPORATIVE POWER OF BOILERS.

W = the actual evaporation;
W_1 = equivalent evaporation from and at 212° F.;
H = total heat of steam above 32° F. at pressure of actual evaporation;
t = observed temperature of feed-water.

$$W_1 = \frac{W(H - t + 32)}{966.1}. \qquad \textbf{(167.)} \quad \text{Art. } \mathbf{927.}$$

QUALITY OF STEAM.

W = weight in pounds of cold water, into which the steam to be tested is run;
w = weight of steam (including any water carried over by it) that is run in;
t = temperature of steam corresponding to the observed pressure;
t_1 = original temperature of the water W;
t_2 = temperature of the mixture after the steam is condensed;

l = latent heat of a pound of steam at the observed pressure;
x = the portion of the weight w that is dry steam;
Q = the quality of the steam $= \frac{x}{w}$.

Then,

$$Q = \frac{x}{w} = \frac{1}{l}\left[\frac{W}{w}(t_2 - t_1) - (t - t_2)\right]. \quad \textbf{(168.)} \quad \text{Art. } \mathbf{929.}$$

FORMULAS USED IN STEAM ENGINES.

WORK DONE BY PISTON.

p = net pressure on piston in pounds per square inch;
V = volume in cubic feet displaced by piston (= area of piston × length of stroke);
W = work done during one stroke.

$$W = 144 p V. \quad \textbf{(169.)} \quad \text{Art. } \mathbf{937.} \quad \S\ 9.$$

REAL AND APPARENT CUT-OFFS.

s = apparent cut-off;
k = real cut-off;
i = the clearance, expressed as a per cent. of stroke.

$$k = \frac{s + i}{1 + i}. \quad \textbf{(170.)} \quad \text{Art. } \mathbf{994.} \quad \S\ 9.$$

SLIP.

P = percentage of slip;
V = velocity of issuing stream in regard to the vessel;
V_1 = velocity of vessel.

$$P = 100 \times \frac{V - V_1}{V}. \quad \textbf{(171.)} \quad \text{Art. } \mathbf{1001.} \quad \S\ 9.$$

CENTER OF PRESSURE IN A PADDLE WHEEL.

P = distance of center of pressure from outer edge of buckets in inches;
a = mean depth in inches of buckets wholly immersed;
b = number of buckets so immersed;
c = mean depth in inches of buckets partially immersed;
d = number of buckets so immersed;
D = diameter of wheel, measured over outer edge of buckets;
D_e = effective diameter.

$$\left.\begin{array}{ll} P = \dfrac{ab+cd}{3(b+d)}. & (a) \\ D_e = D - 2P. & (b) \end{array}\right\} \qquad \textbf{(172.)} \quad \text{Art. } \textbf{1003.} \quad \S\, 9.$$

ROLLING CIRCLE OF PADDLE WHEEL.

S = distance moved by the vessel in feet;
R = number of revolutions of the wheel made while vessel moves through above distance;
D = diameter of rolling circle.

$$D = \frac{S}{3.1416\,R}. \qquad \textbf{(173.)} \quad \text{Art. } \textbf{1008.} \quad \S\, 9.$$

THRUST.

Theoretical Thrust.

W = weight of stream projected from the vessel (whether by paddle, screw, or jet);
V = its velocity, in regard to the surrounding water, in feet per second;
g = 32.16, a constant;
T = the theoretical thrust.

$$T = \frac{WV}{g}. \qquad \textbf{(174.)} \quad \text{Art. } \textbf{1018.} \quad \S\, 9.$$

Indicated Thrust.

H = indicated horsepower of the engines;
P = pitch of screw propeller in feet;
D_e = effective diameter of paddle wheel in feet;
R = revolutions per minute;
T_i = indicated thrust.

For screw steamer:

$$T_i = \frac{33{,}000\, H}{PR}. \qquad \textbf{(175.)} \quad \text{Art. } \textbf{1022.} \quad \S\, 9.$$

For paddle steamer:

$$T_i = \frac{33{,}000\, H}{3.1416\, D_e R} \qquad \textbf{(176.)} \quad \text{Art. } \textbf{1023.} \quad \S\, 9.$$

SPEED OF VESSELS.

Indicated Horsepower and Speed of Vessel.

H = indicated horsepower;
W = displacement of vessel in tons;
k = a constant (see table of Constants for Speed Formulas);
S = speed in knots.

$$H = \frac{S^3 \sqrt[3]{W^2}}{k}. \qquad \textbf{(177.)} \quad \text{Art. } \textbf{1028.} \quad \S\, 9.$$

Revolutions of Engine and Speed of Vessel.

R = revolutions per minute for a given speed;
S = given speed;
R_1 = required revolutions per minute;
S_1 = required speed.

$$R_1 = \frac{R\, S_1}{S}. \qquad \textbf{(178.)} \quad \text{Art. } \textbf{1030.} \quad \S\, 9.$$

HORSEPOWER OF ENGINES.

H_1 = required indicated horsepower;
R_1 = revolutions per minute at the required horsepower;
H = given indicated horsepower;
R = revolutions per minute corresponding to it.

$$H_1 = \frac{R_1^3 H}{R^3}. \qquad (179.) \quad \text{Art. } \mathbf{1032.} \quad \S 9.$$

INDICATED HORSEPOWER.

I. H. P. = indicated horsepower of engine;
P = mean effective pressure (M. E. P.) in pounds per square inch;
A = area of piston in square inches;
L = length of stroke in feet;
N = number of strokes per minute.

$$\text{I. H. P.} = \frac{PLAN}{33{,}000}. \qquad (180.) \quad \text{Art. } \mathbf{1067.} \quad \S 10.$$

MEAN EFFECTIVE PRESSURE.

p = gauge pressure;
k = a constant corresponding to the apparent cut-off (see table of Constants for Apparent Cut-Offs);
M. E. P. = mean effective pressure.

$$\text{M. E. P.} = .9\,[k\,(p + 14.7) - 17]. \qquad (181.)$$

Art. **1068.** § 10.

PISTON SPEED.

l = length of stroke in inches;
R = number of revolutions per minute;
S = piston speed in feet per minute.

$$S = \frac{lR}{6}. \qquad (182.) \quad \text{Art. } \mathbf{1069.} \quad \S 10.$$

MECHANICAL EFFICIENCY OF ENGINE.

I. H. P. = indicated horsepower, or total horsepower developed;

Friction H. P. = horsepower absorbed in overcoming the frictional resistance of engine itself;

Net H. P. = the net horsepower; that is, the horsepower remaining for the performance of useful work = I. H. P. − Friction H. P.;

E_m = mechanical efficiency of engine.

$$E_m = \frac{\text{Net H. P.}}{\text{I. H. P.}}. \qquad \textbf{(183.)} \quad \text{Art. } \mathbf{1073.} \quad \S\,10.$$

STEAM CONSUMPTION.

l = distance in inches between two points on the diagram, one on the expansion line and the other on the compression line, both equally distant from the vacuum line;

a = pressure of steam at the above points;

W = weight of a cubic foot of steam at the above pressure;

L = length of diagram in inches (as $e\,h$, Fig. 320, Art. **1074**);

Q = steam consumption in pounds per I. H. P. per hour.

$$Q = \frac{13{,}750\ l\,W}{PL}. \qquad \textbf{(184.)} \quad \text{Art. } \mathbf{1078.} \quad \S\,10.$$

THERMAL EFFICIENCY OF ENGINE.

T_1 = absolute temperature of steam on entering the cylinder;

T_2 = absolute temperature of steam on leaving the cylinder;

E_t = thermal efficiency.

$$E_t = \frac{T_1 - T_2}{T_1}. \qquad \text{Art. } \mathbf{1081.} \quad \S\,10.$$

WATER REQUIRED FOR CONDENSATION.

t_1 = temperature of departing condensing water;
t_2 = temperature of entering condensing water;
t_3 = temperature of condensed steam upon leaving condenser;
H = total heat of vaporization of one pound of steam at the pressure of the exhaust. (This may be obtained from column 5 of the table of Properties of Saturated Steam.)
W = number of pounds of water required to condense a pound of steam.

$$W = \frac{H - t_3 + 32}{t_1 - t_2}. \qquad \textbf{(185.)} \quad \text{Art. } \mathbf{1096.} \quad \S\ 10.$$

RATIO OF EXPANSION.

e = ratio of expansion in high-pressure cylinder;
E = total ratio of expansion;
v = volume of cylinder receiving steam from boiler;
V = volume of cylinder exhausting into atmosphere or condenser.

$$E = \frac{e\,V}{v}. \qquad \textbf{(186.)} \quad \text{Art. } \mathbf{1103.} \quad \S\ 10.$$

FEED-WATER.

a = numerator of the fraction expressing the saturation at which the boiler is to be worked;
b = net feed-water per stroke in cubic inches;
c = gross feed-water (i. e., the quantity of water per stroke of the feed-pump).

$$c = \frac{a\,b}{a - 1}. \qquad \textbf{(187.)} \quad \text{Art. } \mathbf{1155.} \quad \S\ 10.$$

FORMULAS USED IN RECENT DEVELOPMENTS IN MARINE ENGINEERING.

REFRIGERATING CAPACITY.

F = refrigerating capacity;
H = B. T. U. abstracted in 24 hours.

$$F = \frac{H}{288{,}000}. \qquad \text{Art. } \mathbf{9.} \quad \S\ 12.$$

RULES AND FORMULAS USED IN DYNAMOS AND MOTORS.

DIRECTION OF LINES OF FORCE AROUND A CONDUCTOR.

Rule.—*If the current is flowing in the conductor away from the observer, then the direction of the lines of force will be around the conductor in the direction of the hands of a watch.* Art. **26.** § 28.

TO DETERMINE THE POLARITY OF A SOLENOID.

Rule.—*In looking at the end of the helix, if it is so wound that the current circulates around the helix in the direction of the hands of a watch, that end will be a south pole; if in the other direction, it will be a north pole.* Art. **29.** § 28.

RESISTANCE OF CONDUCTORS.

Let r_1 = the original resistance of a conductor;
r_2 = the changed resistance;
l_1 = the original length;
l_2 = the changed length;
a_1 = the original sectional area;
a_2 = the changed sectional area;
D = the original diameter;
d = the changed diameter;
k = temperature coefficient;
t = rise or fall in temperature, degrees Fahrenheit.

For a change in the length of a conductor:

$$r_2 = \frac{r_1 l_2}{l_1}. \qquad (1.) \quad \text{Art. } \mathbf{40.} \quad \S\ 28.$$

For a change in the sectional area of a conductor:

$$r_2 = \frac{r_1 a_1}{a_2}. \qquad (2.) \quad \text{Art. } \mathbf{41.} \quad \S\ 28.$$

For a change in the diameter of a conductor:

$$r_2 = \frac{r_1 D^2}{d^2}. \qquad \text{(3.) Art. 42. § 28.}$$

For a rise in the temperature of a conductor:

$$r_2 = r_1 (1 + t k). \qquad \text{(4.) Art. 46. § 28.}$$

For a fall in the temperature of a conductor:

$$r_2 = \frac{r_1}{1 + t k}. \qquad \text{(5.) Art. 47. § 28.}$$

RESISTANCES AND TEMPERATURE COEFFICIENTS OF DIFFERENT METALS.

Name of Metal.	Resistance, Microhms per Cu. In.	Relative Resistance.	Temperature Coefficient.
Silver, annealed........	.5921	1.000	.002094
Copper, annealed......	.6292	1.063	.002155
Silver, hard-drawn....	.6433	1.086	.002094
Copper, hard-drawn...	.6433	1.086	.002155
Gold, annealed........	.8102	1.369	.002028
Gold, hard-drawn.....	.8247	1.393	.002028
Aluminum, annealed..	1.1470	1.935	....
Zinc, pressed.........	2.2150	3.741	.002028
Platinum, annealed....	3.5650	6.022	....
Iron, annealed........	3.8250	6.460	....
Nickel, annealed......	4.9070	8.285	....
Tin, pressed..........	5.2020	8.784	.002028
Lead, pressed.........	7.7280	13.050	.002150
German Silver........	8.2400	13.920	.000244
Antimony, pressed....	13.9800	23.600	.002161
Mercury..............	37.1500	62.730	.000400
Bismuth, pressed......	51.6500	87.230	.001967

CURRENT STRENGTH, ELECTROMOTIVE FORCE, AND RESISTANCE.

Let C = strength of current flowing in a closed circuit;
E = electromotive force;
R = resistance.

$$C = \frac{E}{R}. \quad \textbf{(6.)} \text{ Art. } \textbf{61.} \ \S\, 28.$$

$$R = \frac{E}{C}. \quad \textbf{(7.)} \text{ Art. } \textbf{62.} \ \S\, 28.$$

$$E = C R. \quad \textbf{(8.)} \text{ Art. } \textbf{63.} \ \S\, 28.$$

TO FIND THE AVAILABLE ELECTROMOTIVE FORCE IN A CELL.

Let E = the total generated E. M. F.;
E' = *available* E. M. F. when the circuit is closed;
C = the current flowing when the circuit is closed;
r_i = the internal resistance of the cell.

$$E' = E - C r_i. \quad \textbf{(9.)} \text{ Art. } \textbf{67.} \ \S\, 28.$$

THE CURRENT AND RESISTANCE IN BRANCHES OF DIVIDED CONDUCTORS.

Let r_1 = resistance of first branch;
r_2 = resistance of second branch;
r_3 = resistance of third branch;
c_1 = current in first branch;
c_2 = current in second branch;
C = sum of the currents in the two branches;
R'' = joint resistance of two branches in parallel;
R''' = joint resistance of three branches in parallel.

$$c_1 = \frac{C r_2}{r_1 + r_2}. \quad \textbf{(10.)} \text{ Art. } \textbf{69.} \ \S\, 28.$$

$$c_2 = \frac{C r_1}{r_1 + r_2}. \quad \textbf{(11.)} \text{ Art. } \textbf{69.} \ \S\, 28.$$

$$R'' = \frac{r_1 r_2}{r_1 + r_2}. \quad \textbf{(12.)} \text{ Art. } \textbf{71.} \ \S\, 28.$$

$$R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}. \quad \textbf{(13.)} \text{ Art. } \textbf{72.} \ \S\, 28.$$

ELECTRICAL QUANTITY.

Let Q = quantity of electricity in coulombs;
C = current strength in amperes;
t = time in seconds.

$Q = Ct.$ **(14.)** Art. **76.** § 28.

ELECTRICAL WORK AND POWER.

J = electrical work in joules;
F. P. = work in foot-pounds;
Q = quantity of electricity in coulombs;
C = current in amperes;
t = time in seconds during which the current flows;
E = potential, or E. M. F., of circuit;
R = resistance of circuit;
W = power in watts;
H. P. = horsepower.

$J = CEt.$ **(15.)** Art. **78.** § 28.

$J = C^2Rt.$ **(16.)** Art. **78.** § 28.

$J = \frac{E^2t}{R}.$ **(17.)** Art. **78.** § 28.

F. P. $= .7373\,J.$ **(18.)** Art. **79.** § 28.

$W = CE.$ **(19.)** Art. **80.** § 28.

$W = C^2R.$ **(20.)** Art. **80.** § 28.

$W = \frac{E^2}{R}.$ **(21.)** Art. **80.** § 28.

H. P. $= \frac{W}{746}.$ **(22.)** Art. **81.** § 28.

$W =$ H. P. $\times 746.$ **(23.)** Art. **82.** § 28.

TO DETERMINE THE DIRECTION OF THE CURRENT GENERATED IN A CONDUCTOR.

Rule.—*Place thumb, forefinger, and middle finger of the right hand so that each will be perpendicular to the other two; if the forefinger points in the direction of the lines of force and the thumb points in the direction towards which the conductor is moving, then the middle finger will point in the direction towards which the current generated in the conductor tends to flow.* Art. **8.** § 29.

DETERMINATION OF ELECTROMOTIVE FORCE.

Let E = maximum electromotive force obtained at the brushes;

N = total number of lines of force passing from north pole through the core to the south pole;

S = number of outside wires on the periphery through which the current flows *in series;*

n = number of complete revolutions per second of core.

$$E = \frac{2NSn}{10^8}. \qquad (1.)$$ Art. **23.** § 29.

TO DETERMINE THE DIRECTION OF MOTION IMPARTED TO A CONDUCTOR.

Rule.—*Place thumb, forefinger, and middle finger of the left hand each at right angles to the other two; if the forefinger points in the direction of the lines of force and the middle finger points in the direction towards which the current flows, then the thumb will point in the direction of movement imparted to the conductor.* Art. **26.** § 29.

EFFICIENCY OF A DYNAMO.

Let I = input of a dynamo;

O = output;

E = efficiency, per cent.

$$\mathsf{E} = \frac{100 \times O}{I}. \qquad (2.)$$ Art. **60.** § 29.

PER CENT. LOSS IN A DYNAMO.

Let L = per cent. loss;
I = input;
O = output.

$$L = \frac{100\,(I - O)}{I}. \qquad \textbf{(3.)} \quad \text{Art. } \textbf{61.} \quad \S\ 29.$$

When the output and efficiency are given:

$$I = \frac{100 \times O}{\mathbf{E}}. \qquad \textbf{(4.)} \quad \text{Art. } \textbf{62.} \quad \S\ 29.$$

When the input and efficiency are given:

$$O = \frac{I\mathbf{E}}{100}. \qquad \textbf{(5.)} \quad \text{Art. } \textbf{63.} \quad \S\ 29.$$

HORSEPOWER, TORQUE, AND NUMBER OF REVOLUTIONS OF MOTORS.

Let H. P. = horsepower;
T = torque;
S = number of revolutions per minute.

$$\text{H. P.} = .0001904\, TS. \qquad \textbf{(3.)} \quad \text{Art. } \textbf{62.} \quad \S\ 30.$$

$$S = \frac{\text{H. P.}}{.0001904\ T}. \qquad \textbf{(4.)} \quad \text{Art. } \textbf{62.} \quad \S\ 30.$$

$$T = \frac{\text{H. P.}}{.0001904\ S}. \qquad \textbf{(5.)} \quad \text{Art. } \textbf{62.} \quad \S\ 30.$$

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